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Wind Flow Characteristics in the Wakes of Large Wind Turbines

Volume I—Analytical Model Development

W. R. Eberle
Lockheed Missiles and Space Company, Inc.

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National Aeronautics and Space Administration
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NOTICE

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FOREWORD

The material in this report documents the efforts of the first phase of an overall program conducted by Lockheed Missiles and Space Company, Inc., of Huntsville, Alabama, under NASA contract DEN3-29. The first phase of the program called for a computer model of the wind turbine wake, while the second phase dealt with wake measurements on the Mod-0A wind turbine at Clayton, New Mexico.

The computer program of this report is considered an initial calculation procedure for the wake development of wind turbines because additional expansions and refinements are possible to make the program more comprehensive. Areas for further consideration include additional methods for calculating the effects of atmospheric turbulence, non-equal vertical and lateral wake growth rates, and power recovery factor based on real turbine power characteristics.

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1.0 SUMMARY

One of the proposed approaches to reducing the consumption of fossil fuels (in particular, petroleum and natural gas) in the United States is the use of large wind turbines. It is expected that such turbines will be constructed in arrays in large fields of wind turbines. To minimize cost, it is desired to space the turbines as close to each other as possible. However, the turbines must not be spaced so closely that a turbine lies within the wake of another turbine and thereby the output power of the downwind turbine is reduced.

In order to determine the appropriate spacing between turbines, a research program was initiated to characterize the recovery of the wake behind a large wind turbine. The research program had two aspects. The first was the development of an anlytical model of wake recovery downwind of the turbine. The analytical model was developed to calculate the wake properties as functions of the downwind coordinate and to generate wind speed profiles (i.e., wind speed as a function of the lateral coordinate for selected altitudes and wind speed as a function of the vertical coordinate) at selected distances downwind of the turbine. The inputs to the model included wind speed, ambient turbulence, and turbine geometric parameters. The second aspect of the program was an experimental program to measure the wake behind the Mod-0A wind turbine at Clayton, New Mexico. The Lockheed laser Doppler velocimeter was used to make the measurements. This volume contains a description of the analytical model development.

The analytical model development is based upon the results presented by G. N. Abramovich in "The Theory of Turbulent Jets". The term "jet" is used to describe a flow surrounded by another flow of a different velocity, regardless if the velocity of the inner flow is less than or greater than that of the surrounding flow. Although the Abramovich results were taken as the basis for the analytical model for turbine wakes, several modifications of the Abramovich results were necessary. The Abramovich model assumes that the initial jet emanates from an opening. Therefore, it was necessary to add calculations to relate the turbine to the initial wake as presented by Abramovich. Second, the results of Abramovich were developed for flow with no ambient turbulence. Therefore, the effects of ambient turbulence were added to the Abramovich model for the turbine wake model.

The wake growth rate due to ambient turbulence was taken from atmospheric diffusion theory as developed by F. Pasquill. The method of combining the wake growth rate from the Abramovich model with the wake growth rate due to ambient turbulence was taken from the work of P. B. S. Lissaman from a prior model for the wake recovery behind large wind turbines.

Third, the concept of a turbine power factor, which is the ratio of the power generated by a turbine centered in the wake of another turbine to that generated by a turbine in the free stream air, was added to the model.

The mathematics of the wake model was written in a FORTRAN computer program. The FORTRAN computer program was implemented on a PDP-10 computer using the DISPLA graphics package for graphical presentation of results. In addition to the computer program, a version of the wake program was formulated for the TI-59 programmable calculator. The calculator version of the wake program is presented in an appendix.

Several runs of the computer program were made to illustrate the effect of wake variables on wake recovery. The effect of ambient turbulence, initial wind speed deficit in the wake, the height of the turbine rotor hub above the ground, and the power law coefficient for the free stream wind were investigated. The results of these effects are presented in figures in the report.

The principal indicator of wake recovery is the turbine power factor. Much of turbine operation occurs under conditions of neutral atmospheric stability, represented by Pasquill stability class D. For this condition and for an initial velocity ratio of 1.5 (typical for large wind turbines), the turbine power factor is approximately 0.5, 0.75, 0.9, and 0.96 at 10, 20, 35, and 50 rotor radii downwind, respectively.

2.0 INTRODUCTION

One of the proposed approaches to reducing the consumption of fossil fuels (in particular, petroleum and natural gas) in the United States is the use of large wind turbines. It is expected that such turbines will be constructed in arrays in large fields of wind turbines. To minimize cost, it is desired to space the turbines as close to each other as possible. However, the turbines must not be spaced so closely that a turbine lies within the wake of another turbine and thereby the output power of the downwind turbine is reduced.

In order to determine the wake characteristics of turbines, a research program was initiated to characterize the recovery of the wake behind a large wind turbine. The research program had two aspects. The first was the development of an analytical model of wake recovery downwind of the turbine. The second aspect of the program was an experimental effort to measure the wake behind the Mod-0A wind turbine at Clayton, New Mexico. The Lockheed laser Doppler velocimeter (LDV) was used to make the measurements. This report presents the results of the first phase of the investigation, which involved the development of an analytical wake model to calculate the wind speed profile (i.e., wind speed as a function of the lateral coordinate for selected altitudes) at selected distances downwind of the turbine. The principal inputs to the model include wind speed, ambient turbulence, and turbine geometry and operating parameters.

The initial wake behind a wind turbine can be represented as a volume of flow of lower momentum than the surrounding flow. This mass gradually acquires the speed of the surrounding flow as it progresses downwind. Some features are similar to those of a wake behind a bluff body as described in reference 1, while other features are like those of coflowing jets as described in reference 2. There are few published reports on the detailed wake behind wind turbines. Templin (ref. 3) and Crafoord (ref. 4) have made theoretical estimates of large scale effects, that is, without considering detailed flow near each turbine. Some experimental wind tunnel work has been done by Ljungstrom (ref. 5), while more detailed wind tunnel work has been conducted by Builtjes (ref. 6). A comprehensive review of this work is given by Lissaman (ref. 7).

The analytical model presented in this report was developed through an evolutionary process. At the beginning of the contract covering the work reported herein, a subcontract was issued to AeroVironment, Inc. (Lissaman and Walker) for the development of an analytical model and computer program for the wake flow. The model developed by AeroVironment was based on previous work done for the Swedish National Board for Energy Source Development. The report describing the details of the AeroVironment model and calculation procedure is reference 8. The general principles of the model are also described in reference 9, which has a wider distribution than reference 8.

Upon close examination of the AeroVironment model, it was determined that there were certain aspects of the model which could be modified to improve the accuracy of the modeling of the fluid mechanics of the wake. There were also some refinements in the model which were originally requested, but which, in retrospect, should have little influence on the overall results. The effects of these refinements were deemed to be less than the uncertainties of some of the major assumptions of the model. The revised version of the model is contained in this report.

Both versions of the analytical model are based on the theory of coflowing jets as presented by Abramovich in reference 2. The term "jet" is used to describe a flow emanating from an opening into a larger surrounding coflowing flow. The velocity of the jet may be greater or less than that of the surrounding flow. In the application of the coflowing jet to the analysis of wind turbine wakes, ambient turbulence must be considered. Thus, for both the AeroVironment model and the revised model as reported herein, the main task was the addition of the effect of ambient turbulence and the ground plane to the Abramovich model, computerizing the results, and providing automated graphical output for the wake characteristics.

This report contains a description of the analytical model and a comparison with the earlier AeroVironment version. Sample calculations using the analytical model are presented, and the effects of the principal input parameters on wake characteristics of a large wind turbine are explored. Also included in the report are appendixes containing a listing of the computer model, a version of the model for use on the TI-59 programmable calculator, and a discussion of the alternate algorithms considered in the development of the model.

3.0 DESCRIPTION OF WAKE MODEL

3.1 Approach

The mathematical model used for calculating the wind speed profiles in the wakes of large wind turbines is described in this section. The basic concept of the mathematical model is that the wind speed profile in the wake is of different mathematical form in different regions of the wake (as defined later), but this form is uniquely defined in each region at each position downwind of the turbine. For each profile region, knowing the wake radius at the beginning of the region and the wake growth rate in the region, the wake radius can be determined at any desired downwind location. Then, for a given mathematical form of the wind speed profile, the centerline wind speed is uniquely determined by the condition that the momentum deficit is conserved from the initial wake to the downwind location.

Turbulence relationships. - The total effective growth rate of the wake is given by the Pythagorean sum of the growth rate due to the mechanical turbulence (i.e., turbulence generated because of velocity gradients in the flow) and that due to the ambient turbulence. The growth rate due to mechanical turbulence is obtained by the experimental results of Abramovich (Ref. 2), and the growth rate due to ambient turbulence is assumed constant along the wake for a given value of the atmospheric turbulence. The Pythagorean sum (i.e., square root of the sum of the squares) was chosen because the total kinetic energy of the combined turbulence equals the sum of the kinetic energy of the mechanical turbulence and the kinetic energy of the ambient turbulence.

Abramovich (ref. 2) has done extensive work on coflowing jets in fluids. Much of his work has been based on experimental results with no ambient turbulence in the jet or in the surrounding fluid. There are two mechanisms of momentum transfer in the coflowing jets for which Abramovich has presented results. The first mechanism is the result of the viscosity of the air acting on the velocity gradients in the flow. This mechanism of momentum transfer could be represented by the Navier-Stokes equations for laminar flow (although Abramovich does not use the Navier-Stokes equations). The second mechanism of momentum transfer is turbulence generated by velocity gradients in the flow. Because of energy extraction by the turbine rotor, the wind speed of the air flowing through the rotor is reduced, thus creating a stream tube of flow with a wind speed less than that of the surrounding free stream. The velocity gradient across the flow creates turbulence. This creates an eddy viscosity (ref. 10), which increases the momentum transfer in the flow. This mechanism of momentum transfer is much more significant than momentum transfer in laminar flow.

In the following discussion, the parameters calculated from Abramovich are termed as being due to "mechanical turbulence" because it is the gradients in the flow which cause the turbulence which causes the momentum transfer in the cases which Abramovich has studied. This term is used to distinguish the effects which Abramovich has presented in ref. 2 from the effects of ambient turbulence which were not included in the effects which Abramovich included in his analysis.

The wake growth rate due to ambient turbulence has been taken from the theory of diffusion of pollutants by turbulence in the atmosphere. Because of recent regulations related to air quality, the theory of pollutant dispersion has been developed extensively over the past few years. The relationship between the dispersion of pollutants and the transfer of momentum in a wake is given by Abramovich.

Wake Geometry. - The wake of a single turbine is shown in Figure 3-1, as idealized for the unbounded flow (i.e., no effect of the ground). The wake is divided into three regions for the calculations. The wake radius in the first two regions increases linearly with downstream distance at a rate set by the effective turbulence, which is a combination of the mechanical turbulence and the ambient turbulence. The influence of the mechanical turbulence diminishes and asymptotically approaches zero as the wake moves downwind. Eventually, the effect of the mechanical turbulence becomes negligible, and the wake growth is essentially determined by ambient turbulence alone. In order to accurately model wake growth with a diminishing influence of mechanical turbulence, the wake boundary of Region III was calculated by a numerical integration. The details of the geometry and flow in these regions will be discussed in the following sections.

In the initial region (Region I), the potential core is that portion of the wake which has been unaffected by the shear between the wake and the outer (ambient) flow. Region I extends to $X = X_H$, the point at which the shear due to the outer flow has completely eroded the potential core of uniform flow downstream of the extraction disk. Wind speed profiles across the wake in this region are not self-similar at various downstream locations due to the change in relative size of the core flow and the turbulent mixing zone. At the end of Region I, a continuous shear layer-like wind speed profile has completely developed but is represented by a slightly different functional form from that used later in the far wake regime. The transition region, Region II, allows for the smooth transition of the completely developed wind speed profile of Region I to that used in the far wake, which is self-similar for all subsequent downstream locations.

Region I also includes the expansion of the wake from the diameter of the physical extraction disk, R_d, to the expanded slipstream value, R_0, that is, the slipstream expansion due to potential effects. The computer model assumes that this expansion occurs at the station of the disk itself so that the wake develops from $R = R_0$ at $X = 0$. Reasons for this assumption are discussed in a following section.

The wake growth rate is identical for Regions I and II and is given by a combination of ambient and mechanical turbulence as discussed in the following sections. The end of Region II occurs at $X_N = nX_H$, where n is derived from the form of the wind speed profile in Region I and the form of the wind speed profile in Region III as described by Abramovich. The wake radius is calculated at the end of Region I and at the beginning of Region III based upon a momentum balance with the initial wake and the form of the wind speed profiles at each of the respective locations. The downwind extent of Region II is then calculated from the ratio of these radii and the wake growth rate in Region I.

Region III is the far wake. In the initial part of Region III, wake growth is controlled by a combination of mechanical and ambient turbulence. Further downwind, the effect of mechanical turbulence asymptotically approaches zero, and the wake growth is essentially controlled by ambient turbulence alone. In the numerical integration in Region III, wake growth due to mechanical turbulence is never mathematically eliminated. However, its influence asymptotically approaches zero.

Figure 3-1 also shows the notation to be used for the geometrical parameters in the following discussion. Upper case R denotes the wake radius in physical units. Notation for lower case r is similar to that shown in Figure 3-1, but denotes wake radius normalized by the turbine rotor radius, R_d. The normalized radius of the potential core is r_1. The normalized outer radius of the wake at the end of Region k is r_{2k}. The notation, r_2, is used for the normalized outer radius at any general location in the wake. The normalized downwind distance at which Regions I and II end are x_H and x_N, respectively. In the following discussion, upper case U denotes wind speed in physical units. Lower case u denotes wind speed which has been normalized by the free stream wind speed, U_∞.

The following sections describe the computer model. The description follows the calculation procedure of the computer program sequentially, giving derivations of equations used as appropriate. The equations follow the development of Abramovich (ref. 2) closely, and no derivation of equations taken directly from Abramovich is given here. The reader is referred to ref. 2 for derivations of these equations. A flow diagram for the computer program is shown in Figure 3-2. A list of symbols is given in Appendix A, and a program listing is given in Appendix B. The equation number in the following description is underlined when the equation is used directly in the computer program. In the program listing in Appendix B, the equation numbers for equations presented in this section are given.

In addition to the computer program, a version of the wake model was developed for the TI-59 programmable calculator. The TI-59 version of the model is presented in Appendix C. The TI-59 version of the program uses the same equations as those developed in this section for the computer model.

3.2 Data Input

The first step of the program is data input. Table 3-1 shows the input data and card formats for the input data. The definitions of input parameters are given in Table 3-2.

The output of the program consists of a set of plots. The plots consist of the wake radius, the wind speed deficit at the center of the wake, the wind speed at the center of the wake, and the turbine power factor (ratio of the power which would be generated by an identical turbine in the wake of the first turbine to that power which would be generated if the downwind turbine were in the free stream wind) plotted as functions of the downwind coordinate. At the user's option, the geometric parameters may be plotted in physical units or as normalized by the rotor radius. Plots of the wind speed in the wake as a function of the lateral coordinate for eight altitudes for each of the selected downwind locations are also made. At the option of the user, the plots are normalized by the free stream wind speed at the hub altitude or by the free stream wind speed at the altitude at which the plots are made. In addition, a plot showing the vertical wind speed profile at each of the downwind locations of the lateral wind speed profiles is made.

The parameter, σ_θ, is an indication of the atmospheric turbulence. It may be entered in one of two ways. First, it may be entered as the standard deviation of wind direction as measured by tower-mounted anemometers. Alternatively, the atmospheric turbulence may be represented by the Pasquill atmospheric stability class. The stability class varies from class A (highly unstable atmosphere) to class F (highly stable atmosphere). Table 3-3 (taken from ref. 11) provides a key to stability classes.

"Strong" incoming solar radiation corresponds to a solar altitude of greater than 60° with clear skies; "slight" insolation corresponds to a solar altitude from 15° to 35° with clear skies. Cloudiness will decrease incoming solar radiation and should be considered along with solar altitude in determining solar radiation. Incoming radiation that would be strong with clear skies can be expected to be reduced to moderate with broken (5/8 to 7/8 cloud cover) middle clouds and to slight with broken low clouds. These methods will give representative indications of stability over open country or rural areas, but are less reliable for urban areas. This difference is due primarily to the influence of the city's larger surface roughness and heat island effects upon the stability regime over urban areas.

In general, inter turbine spaceing for wind turbines should be based on Pasquill stability class D, which corresponds to neutral atmospheric stability. From Table 3-3, stability classes other than class C and class D occur only when wind speeds are too low for turbine operation near rated power. Since wake recovery will be faster for class C and for class D (because of greater ambient turbulence for class C), class D is the appropriate design condition.

The value of the initial veloctiy ratio, m, to be used as input may be determined from one-dimensional momentum theory in the following manner taken from Ref. 12. Let U_∞ be the free stream wind speed, U_T be the wind speed through the turbine disk, and U_0 be the initial wind speed of the wake (i.e., after expansion due to potential effects as discussed in the following discussion). For the one-dimensional momentum analysis, power is extracted from the disk uniformly over the disk area, A. The axial thrust on the disk is

T = Momentum flux in - Momentum flux out

or

$$T = \dot{m}(U_\infty - U_0) = \rho A U_T (U_\infty - U_0) \qquad (3\text{-}1)$$

where $\dot{m}$ is the mass flow rate of air passing through the turbine disk, and ρ is the mass density of the air. Also, from pressure considerations, the thrust can be expressed as

$$T = A(p^+ - p^-) \qquad (3\text{-}2)$$

where p^+ and p^- are the static pressures on the upwind and downwind sides of the turbine, respectively. Applying the momentum equation (Bernoulli's equation) on the upwind side of the turbine gives

$$p_\infty + \tfrac{1}{2}\rho U_\infty^2 = p^+ + \tfrac{1}{2}\rho U_T^2 \qquad (3\text{-}3)$$

where p_∞ is atmospheric static pressure. On the downwind side of the turbine

$$p^- + \tfrac{1}{2}\rho U_T^2 = p_\infty + \tfrac{1}{2}\rho U_0^2 \qquad (3\text{-}4)$$

Subtracting these equations to get $(p^+ - p^-)$ and using equation (3-2) gives

$$T = A(p^+ - p^-) = \tfrac{1}{2}\rho A(U_\infty^2 - U_0^2) \qquad (3\text{-}5)$$

Equating this with equation (3-1) gives

$$U_T = \tfrac{1}{2}(U_\infty + U_0) \qquad (3\text{-}6)$$

From equation (3-6), it is seen that the wind speed through the turbine is the average of the free stream wind speed ahead of the turbine and the wind speed in the expanded wake of the turbine.

The axial induction factor, a, is defined by

$$U_T \equiv U_\infty(1-a) \qquad (3\text{-}7)$$

Therefore, from equation (3-6),

$$U_0 = U_\infty(1-2a) \qquad (3\text{-}8)$$

and the initial velocity ratio, m, is

$$m \equiv \frac{U_\infty}{U_0} = \frac{1}{1-2a} \qquad (3\text{-}9)$$

The initial velocity ratio can also be expressed in terms of the turbine output power. Because power is given by the mass flow rate times the change in kinetic energy, ΔKE, of the wind flowing through the turbine, the power, P_∞, is

$$P_\infty = \dot{m}(\Delta KE) = \rho A U_T \left[\frac{U_\infty^2}{2} - \frac{U_0^2}{2} \right] \qquad (3\text{-}10)$$

or, substituting for U_T from equation (3-7) and for U_0 from equation (3-8) and simplifying,

$$P_\infty = 2\rho A U_\infty^3 a(1-a)^2 \qquad (3\text{-}11)$$

The power, P_∞, has a maximum at a = 1/3 and a minimum at a = 1. Therefore, if the turbine is operating at its maximum power for the given free stream wind speed, m = 3. If the turbine is not extracting the maximum power available at its free stream wind speed, the appropriate value of "a" must be calculated from equation (3-11). An iterative solution is necessary because equation (3-11) is a cubic equation in "a". The power to be used in equation (3-11) is the power extracted from the wind, not the electric power output of the generator. The aerodynamic power is the power extracted from the air by the turbine blades. For an operating tubine, it may be obtained from the output power and the generator and shafting efficiencies.

After the values of the input parameters have been read, the program writes the values of the input parameters.

3.3 Calculation of Program Constants

Geometric constants for the program are calculated first. The constants include the wake growth rate due to ambient turbulence, the initial wake radius, the downwind extent of Region I, the wake radius at the end of Region I, the downwind extent of Region II, and the wake radius at the end of Region II.

Recalculation of input geometric parameters. - The program allows input of geometric parameters in physical units or in rotor radii. If they have been input in physical units, they are corrected to units of rotor radii.

$$h_a = H_a/R_d \qquad (3\text{-}12)$$

$$z_0 = Z_0/R_d \qquad (3\text{-}13)$$

$$\Delta z = \Delta Z/R_d \qquad (3\text{-}14)$$

$$\Delta y = \Delta Y/R_d \qquad (3\text{-}15)$$

Calculation of the initial wake radius. - From the momentum analysis presented above, the wake expands from the wind speed through the physical extraction disk, U_T, to the initial wake wind speed, U_0. Let R_d be the radius of the turbine disk, and let R_0 be the radius of the wake after expansion to speed, U_0. The mass flow rate of air through the disk is

$$\dot{m} = \rho\pi R_d{}^2 U_T = \rho\pi R_0{}^2 U_0 \qquad (3\text{-}16)$$

From equations (3-6) and (3-9)

$$\frac{R_0{}^2}{R_d{}^2} = \frac{U_T}{U_0} = \frac{(U_\infty + U_0)}{2U_0} = \frac{m+1}{2} \qquad (3\text{-}17)$$

Therefore, the initial wake radius is given in terms of the disk radius as

$$r_0 = \frac{R_0}{R_d} = \sqrt{(m+1)/2} \qquad (3\text{-}18)$$

For the calculation of wake parameters, as indicated previously, the results of Abramovich were derived for mechanically-generated turbulence only (no ambient turbulence). In the following sections, the results of Abramovich are modified to include the effects of ambient turbulence.

Calculation of wake growth due to ambient turbulence. - The program next calculates wake growth due to ambient turbulence. The theory for the turbulent dispersion of plumes (e.g., pollutants) in the atmosphere is used as the basis for the calculation of wake growth due to ambient turbulence. There are three steps in the following discussion. The first step is the determination of the plume growth as measured by the concentration of atmospheric pollutants. The second step relates the concentration of atmospheric pollutants to the wind speed deficit in the wake of turbines. The third step relates the profile parameters for profiles used for pollution concentration work to the profile parameters for profiles used for wakes.

In atmospheric dispersion work, the Gaussian distribution is used to describe the concentration of pollutants. This distribution is given in axisymmetric form as

$$\frac{\chi - \chi_\infty}{\chi_c - \chi_\infty} = \frac{\Delta\chi}{\Delta\chi_c} = e^{-r^2/2\sigma^2} \qquad (3\text{-}19)$$

where χ = concentration of pollutants
χ_c = concentration of pollutants at plume center
χ_∞ = free stream concentration of pollutants (usually zero)
r = radial coordinate
σ = pollution dispersion coefficient.

As mentioned previously, the atmospheric turbulence may be input as the Pasquill stability class or as the standard deviation of wind direction. Input as the Pasquill stability class is considered first. Figure 3-2 of Ref. 11 gives σ as a function of x, the distance downwind of the pollution source, for the six Pasquill stability classes. That figure is reproduced here and shown as Figure 3-3. From this figure, the following values for $d\sigma/dx$ were calculated.

DERIVATIVE OF POLLUTION DISPERSION COEFFICIENT
FOR PASQUILL STABILITY CLASSES

Pasquill stability class	$d\sigma/dx$
A	0.212
B	0.156
C	0.104
D	0.069
E	0.050
F	0.034

Reference 13 relates the Pasquill stability class to σ_θ, the standard deviation of wind direction of the atmosphere. This relation is given as follows.

STANDARD DEVIATION OF WIND DIRECTION FOR PASQUILL STABILITY CLASSES

Pasquill stability class	σ_θ (degrees)
A	25.0
B	20.0
C	15.0
D	10.0
E	5.0
F	2.5

A least squares curve fit of the data given in the two preceding tables gives

$$\frac{d\sigma}{dx} = 0.031e^{.08\sigma_\theta} \qquad (3\text{-}20)$$

Since $\sigma = 0$ at $x = 0$,

$$\sigma(x) = 0.031xe^{.08\sigma_\theta} \qquad (3\text{-}21)$$

If the atmospheric turbulence is input as a Pasquill stability class, table pg. 13 is used to generate a value for $d\sigma/dx$. If the atmospheric turbulence is input as a value of σ_θ, equation (3-20) is used to generate a value for $d\sigma/dx$.

As the second step, it is desired to relate the distribution of concentration of pollution to the distribution of wind speed deficits. In the discussion preceeding equation (5.25), Abramovich states that according to Prandtl's old and new assumptions of free turbulence, the dimensionless profiles of temperature and wind speed are the same, but according to

Taylor's theory of free turbulence, they differ. In the discussion preceeding equation (5.27), Abramovich states that the mechanism of lateral transfer of heat and of admixture is the same; consequently, the profiles of concentration difference must be similar to the profiles of temperature difference. This is stated in equation (5.27) of Abramovich. Combining this result with equation (5.25) of Abramovich gives

$$\frac{\Delta\chi}{\Delta\chi_c} = \sqrt{\frac{\Delta u}{\Delta u_c}} = \sqrt{\frac{U-U_\infty}{U_c-U_\infty}} \qquad (3\text{-}22)$$

where U = wind speed in the wake
U_c = wind speed at the center of the wake
U_∞ = free stream wind speed.

Combining equation (3-22), the result of the Taylor theory mentioned above, with equation (3-19) for a Gaussian wind speed profile

$$\frac{\Delta u}{\Delta u_c} = e^{-2r^2/2\sigma^2} \qquad (3\text{-}23)$$

As a third step, it is desired to relate the σ of the Gaussian wind speed profile given by equation (3-23) to the wake radius, r_2, for the wind speed profile used in the model presented herein. In the far wake, the wind speed profile is given by equation (5.23) of Abramovich as

$$\frac{\Delta u}{\Delta u_c} = \left[1-\left(\frac{r}{r_2}\right)^{1.5}\right]^2 \qquad (3\text{-}24)$$

The wind speed profiles of equation (3-23) and (3-24) are very similar and are compared in Figure 3-4. In determining the relationship between σ and r_2, Δu_c is the same for both profiles, and the mass deficit is the same for both profiles. The equality of mass deficits is

$$2\pi\Delta u_c \int_0^\infty e^{-2r^2/2\sigma^2}\, r\, dr = 2\pi\Delta u_c \int_0^{r_2} \left[1-\left(\frac{r}{r_2}\right)^{1.5}\right]^2 r\, dr \qquad (3\text{-}25)$$

Dividing by $2\pi\Delta u_c$ and integrating between the stated limits gives

$$\frac{\sigma^2}{2} = \frac{9}{70} r_2^{\,2} \qquad (3\text{-}26)$$

or

$$r_2 = 1.97\sigma \qquad (3\text{-}27)$$

Therefore, the wake growth rate due to ambient turbulence is

$$\alpha = (dr_2/dx)_a = 1.97(d\sigma/dx) \qquad (3\text{-}28)$$

where $(d\sigma/dx)$ is obtained from the table on p. 13 or from equation (3-20) according to the method of input of atmospheric turbulence.

The above derivation is based on the Taylor theory of free turbulence. This is the approach used by Abramovich, and Abramovich presents a curve (Figure 5.10 in Abramovich) which shows excellent agreement between experimental data and the Taylor theory. Therefore, the Taylor theory was accepted for use in this model. The Prandtl assumption is stated as

$$\frac{\Delta\chi}{\Delta\chi_c} = \frac{\Delta u}{\Delta u_c} \qquad (3\text{-}29)$$

which is analogous to equation (3-22) for the Taylor theory. Under the Prandtl assumption, equation (3-23) would be

$$\frac{\Delta u}{\Delta u_c} = e^{-r^2/2\sigma^2} \qquad (3\text{-}30)$$

Tracing through the derivation above gives the result

$$r_2 = 2.79\sigma \qquad (3\text{-}31)$$

Therefore, the Prandtl assumption can be used in the model by replacing the 1.97 in equation (3-28) with 2.79.

<u>Calculation of wake characteristics at the end of Region I.</u> - Let U_∞ be the free stream wind speed, U_0 be the initial wind speed in the wake (c.f. equation (3-9)), and U be the local wind speed in the wake. Abramovich assumes the wind speed profile in Region I to be

$$\frac{U_0 - U}{U_0 - U_\infty} = f(\eta) = (1-\eta^{1.5})^2 \qquad (3\text{-}32)$$

where, in the notation of Figure 3-1,

$$\eta = \frac{R_2 - R}{R_2 - R_1} = \frac{r_2 - r}{r_2 - r_1} \qquad (3\text{-}33)$$

Equation (3-32) is based on experimental data.

By definition, the end of Region I is that point at which the potential core vanishes. Thus, the wind speed at the center of the wake at the end of Region I is U_o. A momentum balance between the initial wake and the end of Region I gives

$$r_{21} = \frac{R_{21}}{R_d} = \frac{R_{21}}{R_0}\,\frac{R_0}{R_d} = \frac{r_0}{\sqrt{0.214+0.144m}} \qquad (3\text{-}34)$$

where R_{21}/R_0 is given by equation (5.19) of Abramovich, and r_0 is given by equation (3-18).

From the equation for boundary layer growth given by his equation (5.1), Abramovich gives the length of the initial region of the wake as (equation (5.20))

$$(x_H)_m = \frac{(X_H)_m}{R_0}\,\frac{R_0}{R_d} = \frac{r_0(1+m)}{0.27(m-1)\sqrt{0.214+0.144m}} \qquad (3\text{-}35)$$

The m subscript denotes that the quantity is associated with mechanically-generated turbulence (and is given by the Abramovich model).

Equation (3-34), which gives the wake radius when the potential core has been completely eroded, is derived from conservation of the momentum deficit from the initial wake, the fact that $U = U_0$ at the center of the wake at the end of Region I, and the assumption of the wind speed profile given by equation (3-32). It is therefore valid regardless of the presence or absence of ambient turbulence. Therefore, the presence or absence of ambient turbulence only affects the distance, X_H, at which the end of Region I occurs.

In the development of the model, seven approaches for the downwind extent of Region I were considered. The approaches are described and compared in Appendix D. Two of the approaches are slightly different in concept, but yield identical results. They are physically more justifiable than the other approaches and give numerical results that lie near the middle of the numerical results of all of the other approaches. Therefore, these two approaches have been selected for the model. The reader is referred to Appendix D for a description of the other five approaches.

Figure 3-5(a) shows Region I for the Abramovich solution. The three areas shown are the free stream flow, the potential core, and the boundary layer between the free stream flow and the potential core. Also shown is the line which passes through the initial wake boundary and the midpoint of the wake radius at the end of Region I. The effect of ambient turbulence is shown in figure 3-5(b).

The first approach is based upon the growth of the boundary layer, b. Under this approach, the downwind extent of Region I is defined as that point at which the width of the boundary layer is r_{21}. Since the wake radius ia also r_{21} at the end of Region I and

$$b = r_2 - r_1 \tag{3-36}$$

the radius of the potential core, r_1, must be zero at this point. For mechanical turbulence, the growth rate of the boundary layer is

$$\left(\frac{db}{dx}\right)_m = \frac{r_{21}}{(x_H)_m} \tag{3-37}$$

Furthermore, for this formulation, the ambient turbulence exists in both the free stream flow and in the potential core. Since the ambient turbulence affects both sides of the boundary layer, and α is the wake growth rate due to ambient turbulence on one side of the boundary layer (since it was developed as the rate of growth of the wake radius), the total growth rate of the boundary layer is

$$\frac{db}{dx} = \left[\left(\frac{r_{21}}{(x_H)_m}\right) + (2\alpha)^2\right]^{\frac{1}{2}} \tag{3-38}$$

where the Pythagorean sum of the mechanical turbulence and the ambient turbulence has been used. Since the wake radius is r_{21} at the end of Region I, the downwind extent of Region I is

$$x_H = \frac{r_{21}}{db/dx} = \frac{r_{21}}{\left[\left(\frac{r_{21}}{(x_H)_m}\right)^2 + (2\alpha)^2\right]^{\frac{1}{2}}} \tag{3-39}$$

An alternate approach to the downwind extent of Region I results from the assumption that the boundary layer develops about its own center. For this assumption, the erosion of the inner core, or the growth of the outer radius, is measured relative to the line passing from the initial wake radius to half the wake radius at the end of Region I, as shown in figure 3-5(b). For the erosion of the inner core, let b_1 be the distance from the edge of the potential core to the midpoint of the boundary layer. Hence, for mechanical turbulence,

$$\left(\frac{db_1}{dx}\right)_m = \frac{0.5r_{21}}{(x_H)_m} \tag{3-40}$$

Adding the ambient turbulence from the inner core by the square root of the sum of the squares gives

$$\frac{db_1}{dx} = \left[\left(\frac{0.5r_{21}}{(x_H)_m}\right)^2 + \alpha^2\right]^{\frac{1}{2}} \tag{3-41}$$

Since $b_1 = 0.5r_{21}$ at the end of Region I,

$$x_H = \frac{0.5r_{21}}{\left[\left(\frac{0.5r_{21}}{(x_H)_m}\right)^2 + \alpha^2\right]^{\frac{1}{2}}} \tag{3-42}$$

It is noted that multiplying both the numerator and the denominator of equation (3-42) by 2 gives equation (3-39).

The same result is obtained when the growth of the outer radius of the wake is considered. Let b_2 be the distance from the mid point of the boundary layer to the outer radius of the wake. Then at the end of Region I

$$\left(\frac{db_2}{dx}\right)_m = \frac{0.5r_{21}}{(x_H)_m} \tag{3-43}$$

Adding ambient turbulence gives

$$\frac{db_2}{dx} = \left[\left(\frac{0.5r_{21}}{(x_H)_m}\right)^2 + \alpha^2\right]^{\frac{1}{2}} \tag{3-44}$$

Since $b_2 = 0.5r_{21}$ at the end of Region I,

$$x_H = \frac{0.5r_{21}}{\left[\left(\frac{0.5r_{21}}{(x_H)_m}\right)^2 + \alpha^2\right]^{\frac{1}{2}}} \tag{3-45}$$

which is identical to equation (3-42).

The parameter for the wind speed deficit at the center of the wake is

$$\Delta u_c = \frac{\Delta U_c}{\Delta U_{0c}} = \frac{U_\infty - U_c}{U_\infty - U_0} \tag{3-46}$$

which is the wind speed dificit at the center of the wake divided by the initial wind speed deficit. The c subscript denotes the wind speed in the center of the wake. Because the wind speed in U_0 in the core of the initial region, $\Delta u_c = 1$ in Region I.

Downwind extent and radius at end of Region II. - Region II is a transition region from the wind speed profile form of the initial wake to the wind speed profile form of the far wake. The form of the wind speed profile in Region I is given by equation (3-32). Using this form for the wind speed profile and the condition that the wind speed at the center of the wake is U_0, the wake radius at the end of Region I (denoted by r_{21}) is given by equation (3-34) and is derived from the fact that the momentum deficit at the end of Region I must equal the momentum deficit of the initial wake. As shown later in the discussion of Region III, in Region III, the wind speed profile is of the form given by equation (3-53). Using the same conditions (i.e., wind speed at the center of the wake is U_0 and the momentum deficit of the wake must equal the initial momentum deficit of the wake), the wake radius calculated is greater than r_{21}. Let this wake radius be the wake radius at the end of Region II and be denoted by r_{22}. The downwind extent of Region II is the distance required to allow wake growth from r_{21} to r_{22} at the wake growth rate of Region I. The downwind extent of Region II is given by

$$x_N = nx_H = nX_H/R_d \qquad (3\text{-}47)$$

where n is taken from equation (5.124) in Abramovich as

$$n = \frac{\sqrt{0.214+0.144m}}{1-\sqrt{0.214+0.144m}} \; \frac{1-\sqrt{0.134+0.124m}}{\sqrt{0.134+0.124m}} \qquad (3\text{-}48)$$

The wake growth rate in Region II is identical with that in Region I. Therefore, the wake radius at the end of Region II is given by (cf., Figure 3-1)

$$R_{22}-R_0 = \frac{X_N}{X_H}\,(R_{21}-R_0) \qquad (3\text{-}49)$$

or, dividing by R_d gives

$$r_{22} = \frac{R_{22}}{R_d} = r_0+n(r_{21}-r_0) \qquad (3\text{-}50)$$

The relationship of equation (3-48) is derived solely from the mathematical forms of the wind speed profiles in Region I and Region III and the assumption that the wake growth rate in Region II equals that in Region I. If these same assumptions are made in the presence of ambient turbulence (i.e., the presence of ambient turbulence does not change the form of the wind speed profile in Region I or in Region III, and the wake growth rate in Region II equals that in Region I), then the relationship of equation (3-48) is valid in the presence of ambient turbulence.

3.4 Wake Calculations in Region III

Wake growth in Region III. - Region III of the wake is the region in which the mechanically-generated turbulence decays. Thus, at the beginning of the region, wake growth is governed by both mechanically-generated turbulence and ambient turbulence. The wake growth due to mechanically-generated turbulence asymptotically approaches zero as the downwind coordinate, x, increases. In Region III, the wind speed profiles are self-similar, that is, they have the same mathematical form at all downwind locations. If the applicable expressions from Abramovich are used, the wake growth must be calculated by numerical integration.

Let it be assumed that the wake radius, r_2, is known at distance, x, downwind of the turbine, where r_2 and x are values of the wake radius and downwind distance which have been normalized by the rotor radius, R_d. Let $r_2' = r_2+\Delta r$ be a wake radius which is slightly larger than r_2. It is desired to find the downwind distance at which the wake radius is r_2'.

For the main region of the jet, the unnumbered equation preceeding equation (5.97) of Abramovich gives the wake radius as a function of the wind speed deficit at the center of the wake as

$$r_2 = r_0\left[\frac{n_{2u}-mn_{1u}}{(1-m)^2\left(A_2\Delta u_c{}^2+A_1\left(\frac{m}{1-m}\right)\Delta u_c\right)}\right]^{\frac{1}{2}} \qquad (3\text{-}51)$$

where Δu_c is defined by equation (3-46) and

$$n_{1u} = n_{2u} = 1 \tag{3-52}$$

in the case of uniform fields of velocity and density at the initial cross section of the jet (equation following equation (5.39) of Abramovich).

In the main region of the jet, Abramovich assumes (based on experimental data) that the form of the wind speed profile in the far wake is

$$\frac{\Delta U}{\Delta U_c} = \frac{U_\infty - U}{U_\infty - U_c} = \left[1-\left(\frac{r}{r_2}\right)^{1.5}\right]^2 \tag{3-53}$$

where U_c is the wind speed at the center of the wake. From this assumed form of the wind speed profile, equation (5.86) of Abramovich gives

$$A_1 = 0.258 \tag{3-54}$$

and

$$A_2 = 0.134 \tag{3-55}$$

Therefore, equation (3-51) is

$$r_2 = r_0\left\{(1-m)\left[0.134\Delta u_c^2+\left(\frac{0.258m}{m-1}\right)\Delta u_c\right]\right\}^{-\frac{1}{2}} \tag{3-56}$$

Equation (3-56) is a momentum equation which equates the momentum deficit in the main region of the wake to the initial momentum deficit of the wake. Rearranging equation (3-56) to solve for Δu_c gives

$$0.134\Delta u_c^2-\left(\frac{0.258m}{m-1}\right)\Delta u_c+\frac{r_0^{\,2}}{r_2^{\,2}(m-1)} = 0 \tag{3-57}$$

or

$$\Delta u_c = 3.73\left\{\frac{0.258m}{m-1}-\left[\left(\frac{0.258m}{m-1}\right)^2-\frac{0.536r_0^{\,2}}{r_2^{\,2}(m-1)}\right]^{\frac{1}{2}}\right\} \tag{3-58}$$

where the negative sign for the quadratic equation was chosen so that Δu_c goes to zero as r_2 becomes large.

From his equation (5.28), Abramovich assumes that the wake growth rate is directly proportional to the difference between the wind speed of the free stream flow and the wind speed at the wake center and is inversely proportional to the mean wind speed of the wake. Using the first part of equation (5.31) of Abramovich gives

$$\pm c\frac{dx}{dr_2} = 1+\frac{2U_\infty}{U_c-U_\infty} = 1-\frac{2U_\infty}{\Delta u_c(U_\infty-U_0)} \qquad (3\text{-}59)$$

where the definition of Δu_c given by equation (3-46) has been used in the last term of the equation. Dividing by U_∞, using the definition of m given by equation (3-9) and rearranging gives the wake growth rate due to mechanical turbulence (using $c = -0.27$ for $m > 1$) as

$$\left(\frac{dr_2}{dx}\right)_m = 0.27\left[\frac{2m}{(m-1)\Delta u_c} -1\right]^{-1} \qquad (3\text{-}60)$$

The calculation procedure for wake growth in Region III is as follows. Let it be assumed that the wake radius, r_2, is known at distance, x, downwind of the turbine. Let $r_2' = r_2+\Delta r$ be a wake radius which is slightly larger than r_2. Let $(dr_2/dx)_m$ be the wake growth rate due to mechanically-generated turbulence calculated from equation (3-60) for wake radius, r_2. The value of Δu_c used in equation (3-60) is calculated from equation (3-58). Let $(dr_2/dx)_m'$ be the wake growth rate due to mechanically-generated turbulence for wake radius, r_2'. As before, the wake growth rate due to ambient turbulence is taken as α. Therefore, the total wake growth in the interval Δr between r_2 and r_2' is given by

$$\left(\frac{dr_2}{dx}\right)_e = \left[\left(\frac{(dr_2/dx)_m+(dr_2/dx)_m'}{2}\right)^2+\alpha^2\right]^{1/2} \qquad (3\text{-}61)$$

The downwind distance, x', at which the wake radius is r_2' is given by

$$x' = x+\frac{\Delta r}{(dr_2/dx)_e} \qquad (3\text{-}62)$$

Equations (3-60) through (3-62) are solved iteratively, and r_2 is thereby generated as a function of x. In the calculation procedure, the calculations of equations (3-58) and (3-60) are performed in Subroutine R3 since the calculations must be repeated many times for different values of r_2 and r_2'.

If one of the downwind locations at which wind speed profiles are to be calculated is encountered during the numerical integration of Region III, the wake radius at that point is retained. The wake radius at the desired value of x is obtained by linear interpolation between the values of x which bracket the desired value of x. The numerical integration is continued downwind until $x > 50$ or $r_2 > 7$, whichever occurs first.

Turbine power factor. - As mentioned previously, Subroutine R3 is used to perform the calculations of equations (3-58) and (3-60). The subroutine also calculates the turbine power factor which is defined as

$$\overline{P} = P/P_\infty \qquad (3\text{-}63)$$

where P = power which is generated by a turbine which is geometrically identical to another turbine and centered in the wake of the other turbine

P_∞ = power generated by the turbine in the free stream wind.

The geometry related to a wind turbine in the wake of another wind turbine is shown in Figure 3-6. This geometry represents a worst case condition because the downwind turbine is centered in the wake of the upwind turbine. For actual turbines, the wake oscillates because of changes in the wind direction. However, in the present analysis, it is asssumed that the downwind turbine is fixed in the center of the wake of the upwind turbine.

Immediately upwind of the turbine, the stream tube of the flow passing through the turbine expands from a radius, R_∞, and wind speed U_∞, to the disk radius, R_d, and the wind speed, U_T, through the turbine disk. It is assumed that the distance, ΔX_p, over which the potential effects occur (cf. equations (3-1) through (3-6)) is small compared with the distance between the turbines. Let A_∞ be the cross-sectional area of the stream tube in the free stream. Let the turbine power for a turbine in the free stream air be

$$P_\infty = KA_\infty U_\infty^3 \qquad (3\text{-}64)$$

where, from comparison with equation (3-11), the constant, K, is

$$K = 2\rho Aa(1-a)^2/A_\infty \tag{3-65}$$

For a turbine in the wake of another turbine, equation (3-64) must be modified to show the variation of the "free stream" wind speed across the stream tube for the downwind turbine. For a turbine in the wake of another turbine, the power is

$$P = 2\pi K \int_0^{R_\infty} [U(r)]^3 R \, dR \tag{3-66}$$

or, normalizing R by R_d and normalizing U by U_∞ gives

$$P = 2\pi K U_\infty^3 R_d^2 \int_0^{r_\infty} [u(r)]^3 r \, dr \tag{3-67}$$

Combining the definition of the turbine power factor from equation (3-63) with equations (3-64) and (3-66) gives

$$\overline{P} = \frac{P}{P_\infty} = \frac{2\pi K U_\infty^3 R_d^2 \int_0^{r_\infty} [u(r)]^3 r \, dr}{\pi K R_\infty^2 U_\infty^3} \tag{3-68}$$

or

$$\overline{P} = \frac{2}{r_\infty^2} \int_0^{r_\infty} [u(r)]^3 r \, dr \tag{3-69}$$

From the conservation of mass in the stream tube upwind of the turbine, the mass flow rate of air through the turbine is

$$\dot{m} = \pi \rho R_d^2 U_T = \pi \rho R_\infty^2 U_\infty \tag{3-70}$$

Using equation (3-6) for U_T and rearranging gives

$$R_\infty^2 = \frac{U_\infty + U_0}{2U_\infty} R_d^2 \qquad (3\text{-}71)$$

Dividing by $R_d^2 U_\infty$ and using the definition of m given in equation (3-9) gives

$$r_\infty = \sqrt{\frac{m+1}{2m}} \qquad (3\text{-}72)$$

The preceeding discussion has been related to the downwind turbine (i.e., the turbine which lies within the wake of the upwind turbine). The wake of the upwind turbine forms the free stream wind flow for the downwind turbine. In the far wake (Region III), the form of the wind speed profile is given by equation (3-53). Combining this equation with the definition of Δu_c given in equation (3-46) and the definition of m given in equation (3-9) and dividing by U_∞ gives

$$u = 1 - \Delta u_c \left(1 - \frac{1}{m}\right)\left[1 - \left(\frac{r}{r_2}\right)^{1.5}\right]^2 \qquad (3\text{-}73)$$

If the center of the downwind turbine coincides with the center of the wake with the wind speed profile given by equation (3-73), the turbine power factor is

$$\overline{P} = \frac{2}{r_\infty^2}\int_0^{r_\infty} \left\{1 - \Delta u_c \left(1 - \frac{1}{m}\right)\left[1 - \left(\frac{r}{r_2}\right)^{1.5}\right]^2\right\}^3 r\, dr \qquad (3\text{-}74)$$

For convenience in the derivation, let

$$B = \Delta u_c \left(1 - \frac{1}{m}\right) \qquad (3\text{-}75)$$

Then

$$\overline{P} = \frac{2}{r_\infty^2}\int_0^{r_\infty} \left\{1 - 3B\left[1 - \left(\frac{r}{r_2}\right)^{1.5}\right]^2 + 3B^2\left[1 - \left(\frac{r}{r_2}\right)^{1.5}\right]^4 - B^3\left[1 - \left(\frac{r}{r_2}\right)^{1.5}\right]^6\right\} r\, dr \qquad (3\text{-}76)$$

$$\overline{P} = \frac{2}{r_\infty^2}\int_0^{r_\infty}\left\{[1-3B+3B^2-B^3] + [6B-12B^2+6B^3]\left(\frac{r}{r_2}\right)^{1.5}\right.$$

$$+ [-3B+18B^2-15B^3]\left(\frac{r}{r_2}\right)^{3} + [-12B^2+20B^3]\left(\frac{r}{r_2}\right)^{4.5}$$

$$\left. + [3B^2-15B^3]\left(\frac{r}{r_2}\right)^{6} + 6B^3\left(\frac{r}{r_2}\right)^{7.5} - B^3\left(\frac{r}{r_2}\right)^{9}\right\} r\,dr$$

(3-77)

Integrating between the indicated limits and simplifying gives

$$\overline{P} = 2\left\{\frac{1-3B+3B^2-B^3}{2} + \frac{6B-12B^2+6B^3}{3.5}\left(\frac{r_\infty}{r_2}\right)^{1.5} + \frac{-3B+18B^2-15B^3}{5}\left(\frac{r_\infty}{r_2}\right)^{3}\right.$$

$$\left. + \frac{-12B^2+20B^3}{6.5}\left(\frac{r_\infty}{r_2}\right)^{4.5} + \frac{3B^2-15B^3}{8}\left(\frac{r_\infty}{r_2}\right)^{6} + \frac{6B^3}{9.5}\left(\frac{r_\infty}{r_2}\right)^{7.5} - \frac{B^3}{11}\left(\frac{r_\infty}{r_2}\right)^{9}\right\} \quad \text{(3-78)}$$

The power factor $\overline{P}$ can then be expressed explicitly as a function of m and r_2 by evaluating B from equation (3-58) in conjunction with equation (3-18), and r_∞ from equation (3-72). The expression for B is

$$B = 3.731\left\{0.258 - \left[0.06656 - \frac{0.268}{r_2^{\,2}}\left(1 - \frac{1}{m^2}\right)\right]^{1/2}\right\}$$

Figure 3-7 shows calculated values of $\overline{P}$ as a function of r_2 for three values of m as obtained from equation (3-78) and the above expression for B. This plot is not generated during the computer run.

This analysis was developed for a wake out of ground effect. That is, there is no effect of the ground plane and no wind shear because of the ground. Two other assumptions are involved. The first is that no significant wake growth occurs over the distance of the expansion of the stream tube from radius R_∞ to the disk radius R_d for the downwind turbine. The second assumption is that the axial induction factor, a, is the same for both turbines. Because the turbine power factor is based upon the form of the wind speed profile that exists in Region III, the turbine power factor is undefined in Regions I and II.

3.5 Wake Plots

As mentioned previously, the numerical integration in Region III is terminated when $x > 50$ or $r_2 > 7$, whichever occurs first. Four plots are then made. The first plot shows the wake radius, the wake width, and the height of the top of the wake above the ground as a function of the downwind coordinate, x. The wake width is

$$w = 2r_2 \qquad (3\text{-}79)$$

The height of the top of the wake above the ground is

$$h = r_2 + h_a \qquad (3\text{-}80)$$

The second plot shows Δu_c as a function of x. The third plot shows the normalized wind speed at the center of the wake, u_c. This is obtained from rearranging equation (3-46) to give

$$U_c = U_\infty - \Delta u_c (U_\infty - U_0) \qquad (3\text{-}81)$$

and then dividing by U_∞ to give

$$\frac{U_c}{U_\infty} = u_c = 1 - \Delta u_c \left(1 - \frac{1}{m}\right) \qquad (3\text{-}82)$$

The fourth plot shows the turbine power factor as a function of the downwind coordinate, x, in Region III.

3.6 Calculation of Wind Speed Profiles

After the plots described above have been completed, plots of wind speed profiles are made. The first set of wind speed profiles shows the wind speed as a function of the lateral coordinate for eight altitudes specified by the input values of z_0 and Δz. Plots are made for each of the input desired downwind locations. A plot of the vertical wind speed profile at the wake centerline is then made. A single plot shows the vertical wind speed profile at all of the downwind locations at which lateral wind speed profiles were made.

Coordinates. - If the downwind locations at which wind speed profiles are desired have been input in physical units, they are converted to units of rotor radii by

$$x_{dj} = X_{dj}/R_d \qquad (3\text{-}83)$$

where the notation x_{dj} denotes the downwind coordinate of the jth location at which wind speed profiles are desired.

During the calculation of the wake radius as previously described, the wake radius corresponding to each x_{dj} is calculated and retained. Let r_{sj} be the wake radius at $x = x_{dj}$. For the x_{dj} which are less than x_N

$$r_{sj} = r_0 + (r_{22} - r_0)x_{dj}/x_N \qquad (3\text{-}84)$$

For the x_{dj} which are larger than x_N, the values of r_{sj} are retained as the values of the x_{dj} are determined during the numerical integration of Region III. The wake radius at x_{dj} is obtained by linear interpolation between the values of x^{dj} which were calculated in the numerical integration and which bracket x_{dj}.

The generation of the lateral wind speed profiles begins at $z = z_0$ and $y = 0$ (axis of turbine). Values of the wind speed in the wake are calculated at increments, Δy, beginning at $y = 0$. Once the value of y is in the free stream (indicated by $u = 1$), the free stream value of the wind speed is extended across the plot to $y = 6$ rotor radii. The altitude is then incremented by Δz, and the wind speed profile at the next altitude is generated. The process is repeated until wind speed profiles have been generated for eight altitudes. Since the lateral profiles are symmetrical about $y = 0$, the wind speed for only positive values of y are determined.

After the lateral wind speed profiles have been generated for a given downwind location, x_{dj}, the vertical wind speed profile is generated. The vertical wind speed profile is calculated at $y = 0$ only. The profile begins at $z = 0$ with increments of the input value of Δy used for successive calculations (i.e., $\Delta z = \Delta y$ for this calculation).

The altitude of the point on the wake profile (y,z) relative to the axis of the turbine, denoted by z_v, is also calculated. It is

$$z_v = z - h_a \qquad (3\text{-}85)$$

where z is the altitude as measured from the ground.

Subroutine CALCU is used to calculate the wind speed in the wake for given values of y and z_v. The subroutine first calculates the radius of the point (y, z_v) as

$$r = \sqrt{y^2 + z_v^2} \tag{3-86}$$

Region I. - If $x_{dj} \leq x_H$, x_{dj} is in Region I. The radius of the potential core is

$$r_1 = r_0(1 - x_{dj}/x_H) \tag{3-87}$$

Consider η as defined by equation (3-33) where $r_2 = r_{sj}$. If $\eta > 1$, r is inside the potential core and

$$u = 1/m \tag{3-88}$$

If $\eta < 0$, r is outside the wake boundary, and

$$u = 1 \tag{3-89}$$

If $0 \leq \eta \leq 1$, r is in the boundary layer. Rearranging equation (3-32), dividing by U_∞, and using the definition of m from equation (3-9) gives

$$u = \frac{U}{U_\infty} = \frac{1}{m} + \left(1 - \frac{1}{m}\right)\left(1 - \eta^{1.5}\right)^2 \tag{3-90}$$

After the value of u is calculated, control is returned to the main program.

Region III. - If $x_{dj} \geq x_N$, x_{dj} is in Region III. If $r > r_2$,

$$u = 1 \tag{3-91}$$

If $r < r_2$, Subroutine R3 is called to calculate Δu_c from equation (3-58). Then, with $(U_\infty - U)$ from equation (3-53) and $(U_\infty - U_c)$ from equation (3-46),

$$U_\infty - U = \Delta u_c (U_\infty - U_0)\left[1 - \left(\frac{r}{r_2}\right)^{1.5}\right]^2 \tag{3-92}$$

Dividing by U_∞ and using the definition of m in equation (3-9) gives

$$u = 1-\Delta u_c\left(1-\frac{1}{m}\right)\left[1-\left(\frac{r}{r_2}\right)^{1.5}\right]^2 \qquad (3\text{-}93)$$

For this equation it is seen that if $r = r_2$, $u = 1$. If $r = 0$,

$$u = 1-\Delta u_c\left(1-\frac{1}{m}\right) \qquad (3\text{-}94)$$

which agrees with the definition of Δu_c given in equation (3-46).

Region II. - If $x_H < x_{dj} < x_N$, x_{dj} is in the transition region. The wind speed is linearly interpolated between the profile at the end of Region I and the profile at the beginning of Region III. The actual radius of the wake in Region II is used. Let u_I be the value of u calculated from equations (3-33) and (3-90) using the value of r/r_2 in Region II. Let u_{III} be the value of u calculated from equation (3-93) using the same value of r/r_2. At the beginning of Region III, $\Delta u_c = 1$. Then, for Region II,

$$u = \left(\frac{x_{dj}-x_H}{x_N-x_H}\right)u_{III}+\left(\frac{x_N-x_{dj}}{x_N-x_H}\right)u_I \qquad (3\text{-}95)$$

After calculation of u, control is returned to the main program.

3.7 Calculation with Ground Effect

The above equations describe the wind speed in the wake for an isolated turbine. The effect of the ground is to shield the lower part of the wake from the effect of the ambient wind which would otherwise act to accelerate the flow in the wake. The effect of the ground thus retards the acceleration of the wake flow by the surrounding free stream. Thus, in the presence of the ground effect, the wind speed in the wake is less than it would be if the ground were not present.

The presence of the ground is modeled by placing an image turbine at distance, h_a, below the ground. The imaging technique is shown in Figure 3-8. Let z_v* denote the altitude of the point (y,z), relative to the axis of the image turbine. Then

$$z_v^* = z+h_a \quad (3\text{-}96)$$

Using the value of z_v* instead of z_v, Subroutine CALCU is called to calculate u*, the normalized wind speed at point (y,z) in the wake of the image turbine. The total wind speed deficit in the wake in ground effect is the sum of the wind speed deficits in the wakes of the real turbine and of the image turbine. If u_g is the normalized wind speed for a wake in ground effect, then the sum of the wind speed deficits is

$$1-u_g = (1-u)+(1-u^*) \quad (3\text{-}97)$$

where the 1 is the normalized wind speed of the free stream. Rearranging equation (3-98) gives

$$u_g = u+u^*-1 \quad (3\text{-}98)$$

By definition, the approach conserves the total mass deficit. The mass deficit in the real wake is (1-u). The mass deficit in the wake of the image turbine is (1-u*), and the mass deficit of the wake in ground effect is $(1-u_g)$. Equation (3-97) shows that the mass deficit for the wake in ground effect is the sum of the mass deficits for the wakes of the real and image turbines.

It is noted that the presence of the ground does not affect the shape of the wake boundary above the ground. This is shown in Figure 3-9. The only portion of the wake which is affected by the ground effect is that portion of the wake which lies in the intersection of the wake of the real turbine and the wake of the image turbine.

3.8 Calculation of Wind Speed

For the horizontal wind speed profiles, the input parameter, NP, specifies whether the wind speed is to be normalized by the free stream wind speed at the altitude of the profile or is to be normalized by the free stream wind speed at the hub altitude. If NP = 0, the wind speed in the wake is normalized by the free stream wind speed at the altitude of the wind speed profile. The parameter, u_g, given by equation (3-98) has been developed as the normalized wind speed in the wake in a uniform free stream wind speed. Therefore, $u_g = 1$ in the free stream. For a nonuniform free stream wind speed profile, if the wind speed in the wake is normalized by the free stream wind speed at the altitude of the wind speed profile, the wind speed in the free stream is 1. Therefore, u_g is the wind speed in the wake normalized by the free stream wind speed at the altitude of the wind speed profile.

In NP = 1, the wind speed in the wake is normalized by the free stream wind speed at the hub altitude. Then, the wind speed at the altitude of the wind speed profile is

$$U_g = U_\infty u_g (z/h_a)^\gamma \qquad (3\text{-}99)$$

where $U_\infty (z/h_a)^\gamma$ is the free stream wind speed at the altitude of the wind speed profile, and (as indicated above) u_g is the wind speed in the wake normalized by the free stream wind speed at the altitude of the profile.

If U_∞ has been input in physical units, the graphical output of the program will be in physical units of wind speed instead of normalized by the free stream wind speed. In this case, the use of NP = 1 will give output in physical units of wind speed for each altitude. The output for NP = 0 has no desirable interpretation if $U_\infty \neq 1$.

For the vertical wind speed profile, equation (3-99) is used, regardless of the value of NP or whether the output is normalized by the free stream wind speed or given in physical units. Thus, the wind speed profile of the free stream is always evident on the vertical wind speed profile is $\gamma > 0$.

Table 3-1

FORMAT OF INPUT PARAMETERS FOR TURBINE WAKE COMPUTER PROGRAM

Columns	1-10	11-20	21-30	31-40	41-50	51-60	Format
Card 1	ST	CPM	AH	VH0	WEXP	RRR	6F10.4
Card 2	J, IO, NP						3I2
Card 3	XNPT(1)	XNPT(2)	XNPT(3)	XNPT(4)	XNPT(5)	XNPT(6)	6F10.4
Card 4	Z0	DZZ	DYY				3F10.4

Table 3-2

TABLE OF INPUT PARAMETERS FOR TURBINE WAKE COMPUTER PROGRAM

Symbol	Computer Symbol	Definition
σ_θ	ST	Atmospheric turbulence parameter. If input as a positive number, it is the standard deviation of the wind direction. If input as a negative number, it is the Pasquill stability class as follows: -1 for Pasquill stability class A -2 for Pasquill stability class B -3 for Pasquill stability class C -4 for Pasquill stability class D -5 for Pasquill stability class E -6 for Pasquill stability class F If the input value is 0., the wake growth due to ambient turbulence is zero, and the Abramovich wake solution results.
m	CPM	Ratio of free stream wind speed, U_∞, to the initial wind speed in the wake, U_0.
h_a or H_a	AH	Hub height of the turbine in rotor radii or in physical units as specified by IO.
U_∞	VHO	Ambient wind speed at the hub altitude. If output is desired in physical units, input value should be in physical units. If output normalized by free stream value is desired, input should be 1.0.
γ	WEXP	Coefficient of the power law profile for the free stream wind speed.

Table 3-2

TABLE OF INPUT PARAMETERS FOR TURBINE WAKE COMPUTER PROGRAM (Continued)

Symbol	Computer Symbol	Definition
R_d	RRR	Rotor radius. An input value of 1.0 will give all output of units of length in rotor radii. An input value of 0.5 will give all output of units of length in rotor diameters. An input value greater than 2.0 will give all output of units of length in the physical units used for the rotor radius.
J	J	Number of downwind locations at which wind speed profiles are to be calculated.
IO	IO	Input option for parameters with physical dimensions of length (AH, XNPT, ZO, DZZ, DYY) IO = 1 for input in rotor radii IO = 2 for input in physical units (must be same physical units as used for rotor radius).
NP	NP	Specifies how velocity is to be normalized for plots of wind speed profiles in the lateral direction. NP = 0 normalizes wind speed by the free stream wind speed at that altitude NP = 1 normalizes wind speed by the free stream wind speed at the hub altitude.
x_d or X_d	XNPT(j)	Downwind locations at which wind speed profiles are to be calculated (rotor radii or physical units as specified by IO).

Table 3-2

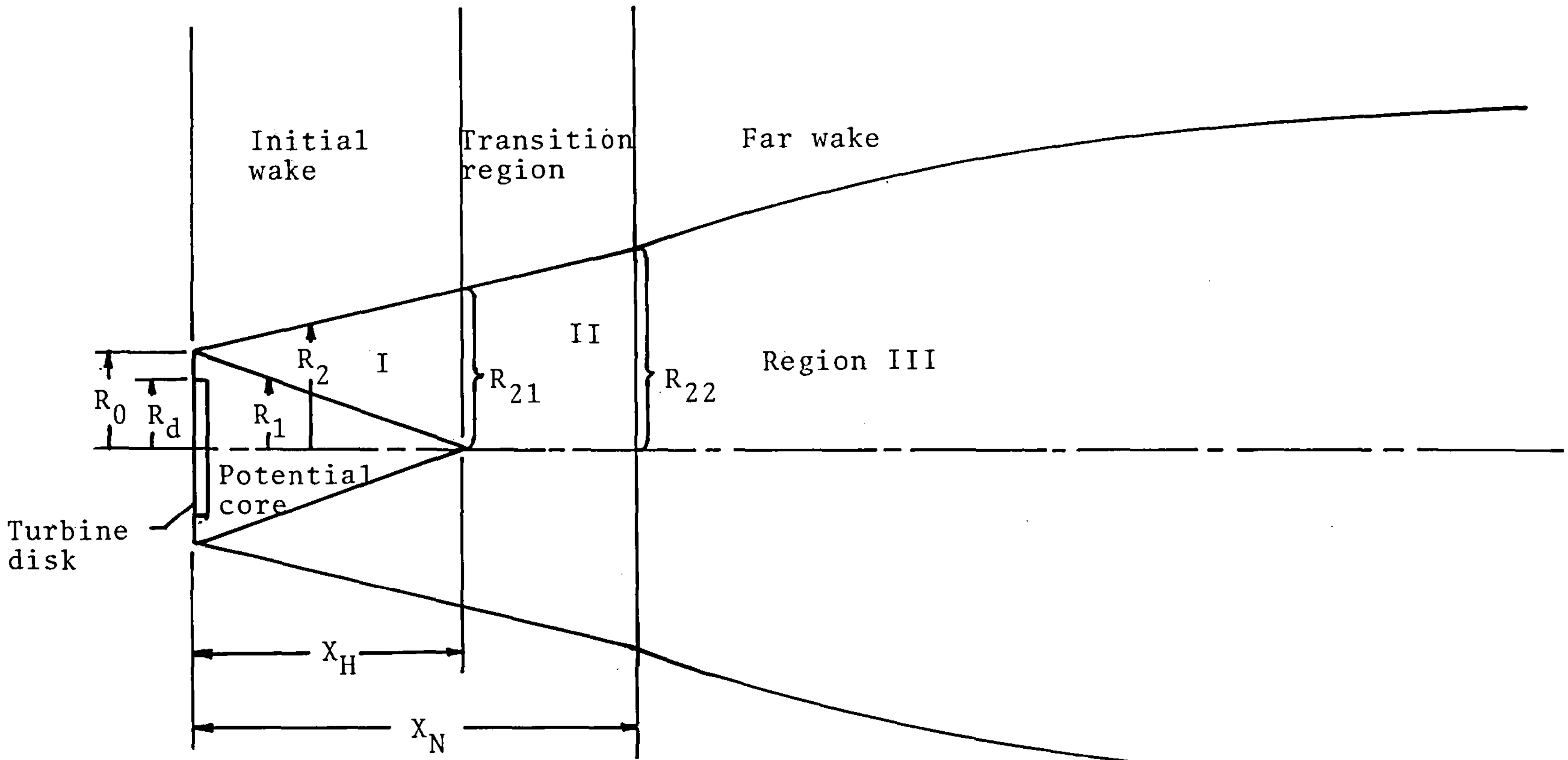
TABLE OF INPUT PARAMTERS FOR TURBINE WAKE COMPUTER PROGRAM (Concluded)

Symbol	Computer Symbol	Definition
z_0 or Z_0	ZO	Minimum altitude at which wind speed profiles are to be calculated (rotor radii or physical units as specified by IO).
Δz or ΔZ	DZZ	Increment in altitudes at which wind speed profiles are to be calculated (rotor radii or physical units as specified by IO).
Δy or ΔY	DYY	Increment in y by which calculations are to by made in generating the wind speed profiles (rotor radii or physical units as specified by IO).

Table 3-3

KEY TO PASQUILL STABILITY CALSSES

Surface wind speed at 10m (m/sec)	Day: Incoming solar radiation			Night	
	Strong	Moderate	Slight	Thinly overcast or ≥4/8 Low cloud	≤3/8 Cloud
<2	A	A-B	B		
2-3	A-B	B	C	E	F
3-5	B	B-C	C	D	E
5-6	C	C-D	D	D	D
>6	C	D	D	D	D



R_d = Radius of turbine disk

R_0 = Initial wake radius

R_1 = Radius of potential core

R_2 = Outer radius of wake

R_{21} = Outer radius of wake at end of Region I

R_{22} = Outer radius of wake at end of Region II

X_H = Downwind extent of Region I

X_N = Downwind extent of Region II

Figure 3-1. - Wake geometry for the wake computer model.

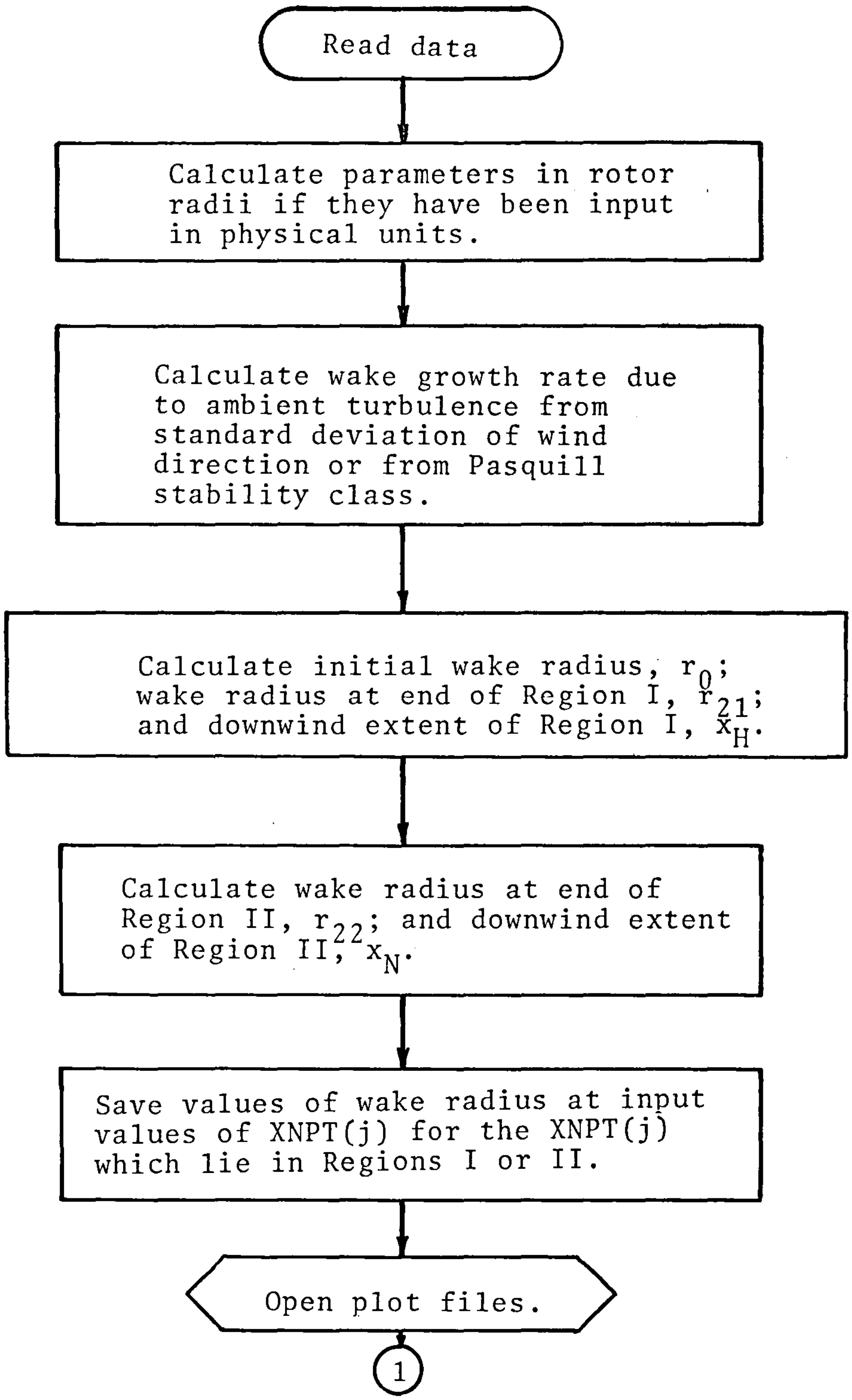


Figure 3-2. Flow diagram for wake model computer program.

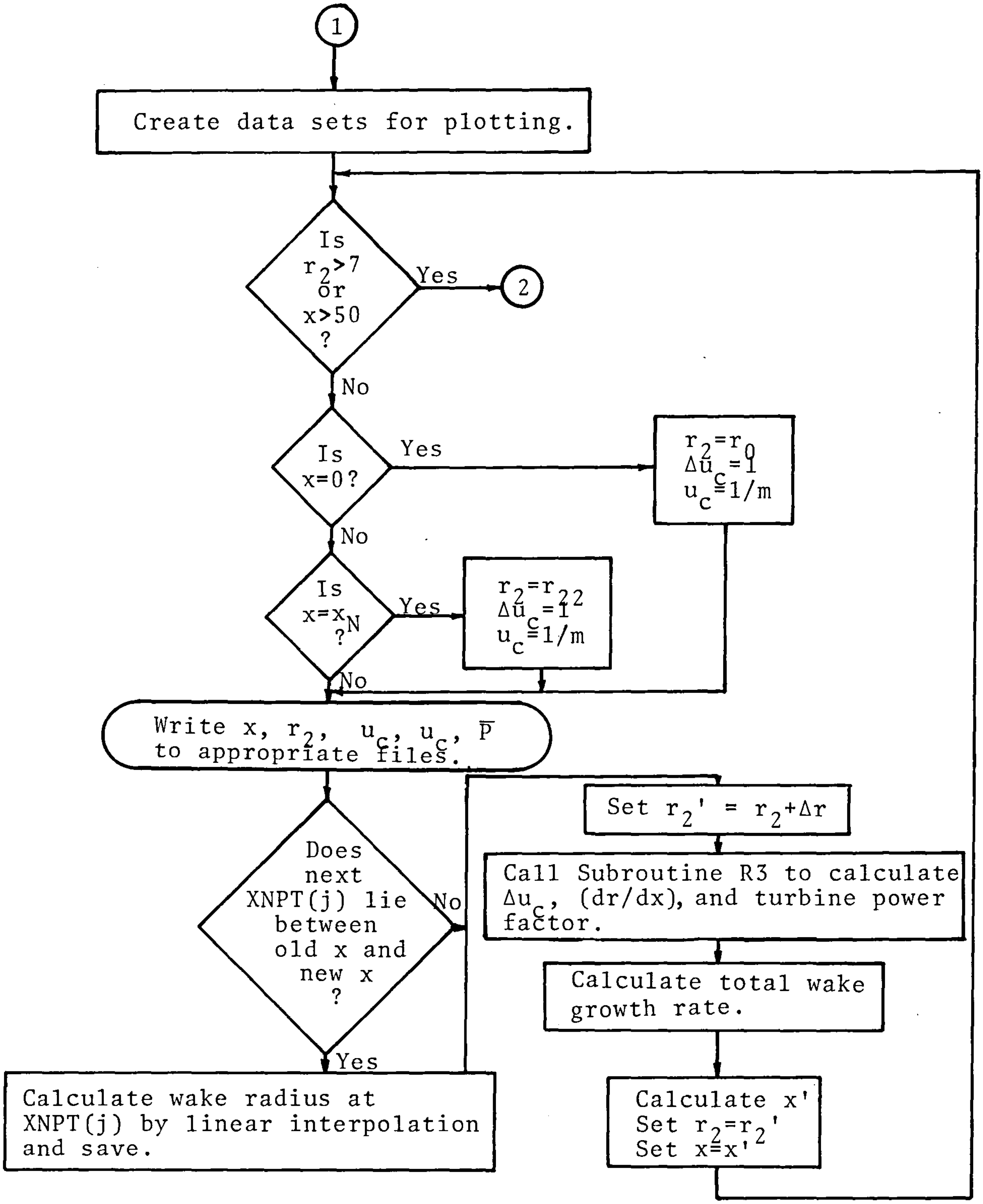


Figure 3-2. Flow diagram for wake model computer program (continued).

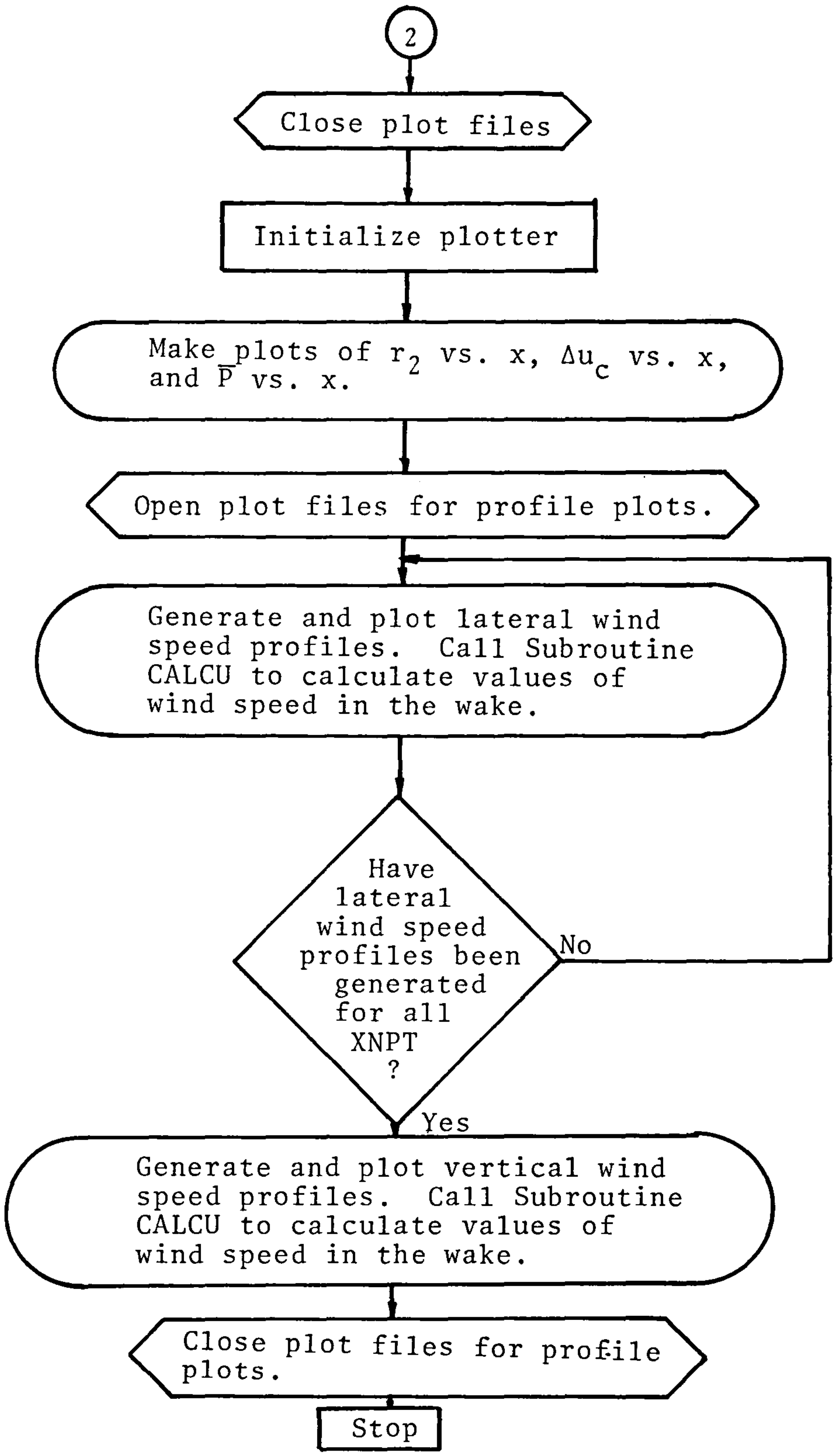


Figure 3-2. Flow diagram for wake model computer program (concluded).

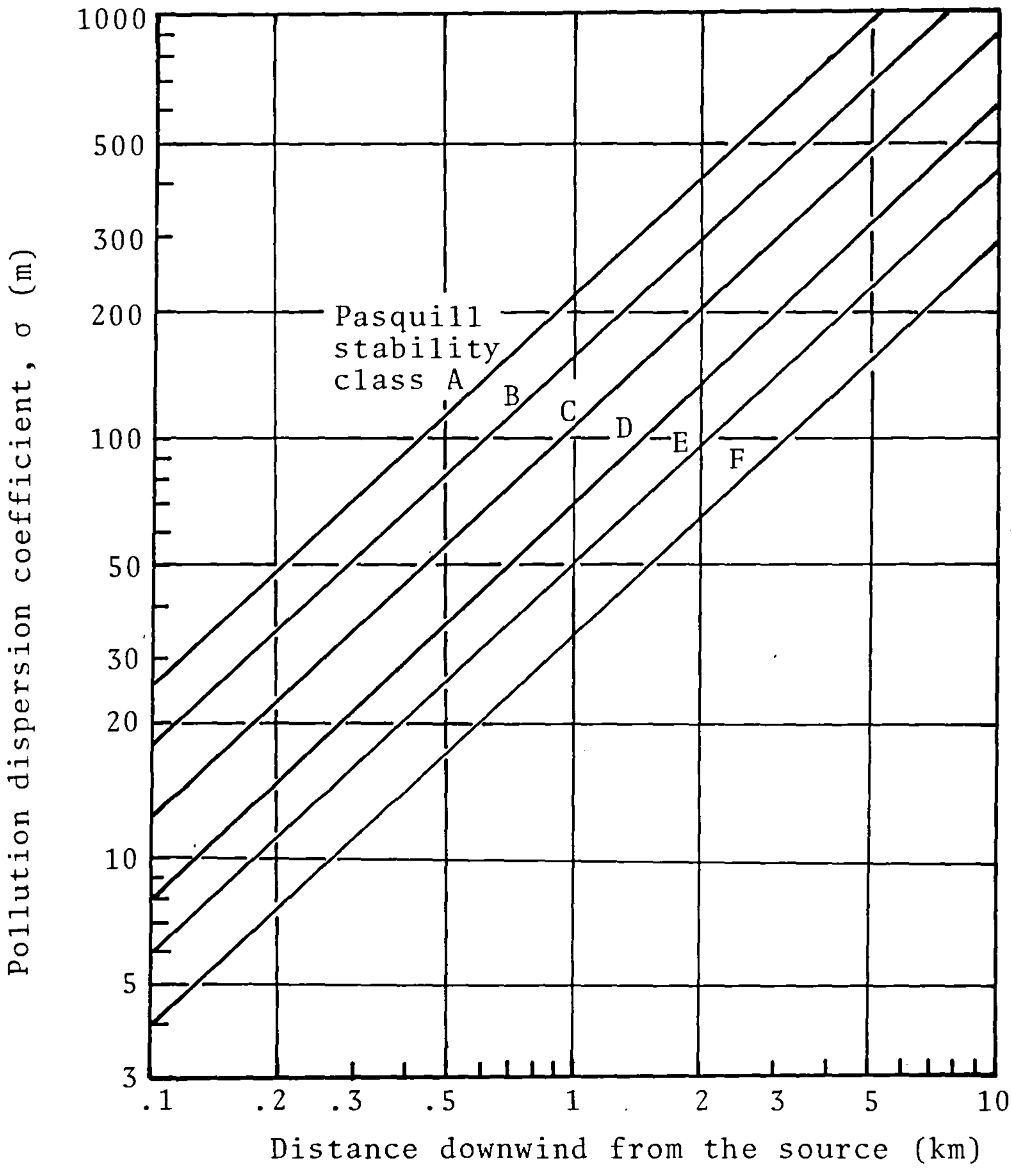


Figure 3-3. Pollution dispersion coefficient as a function of distance downwind of the source.

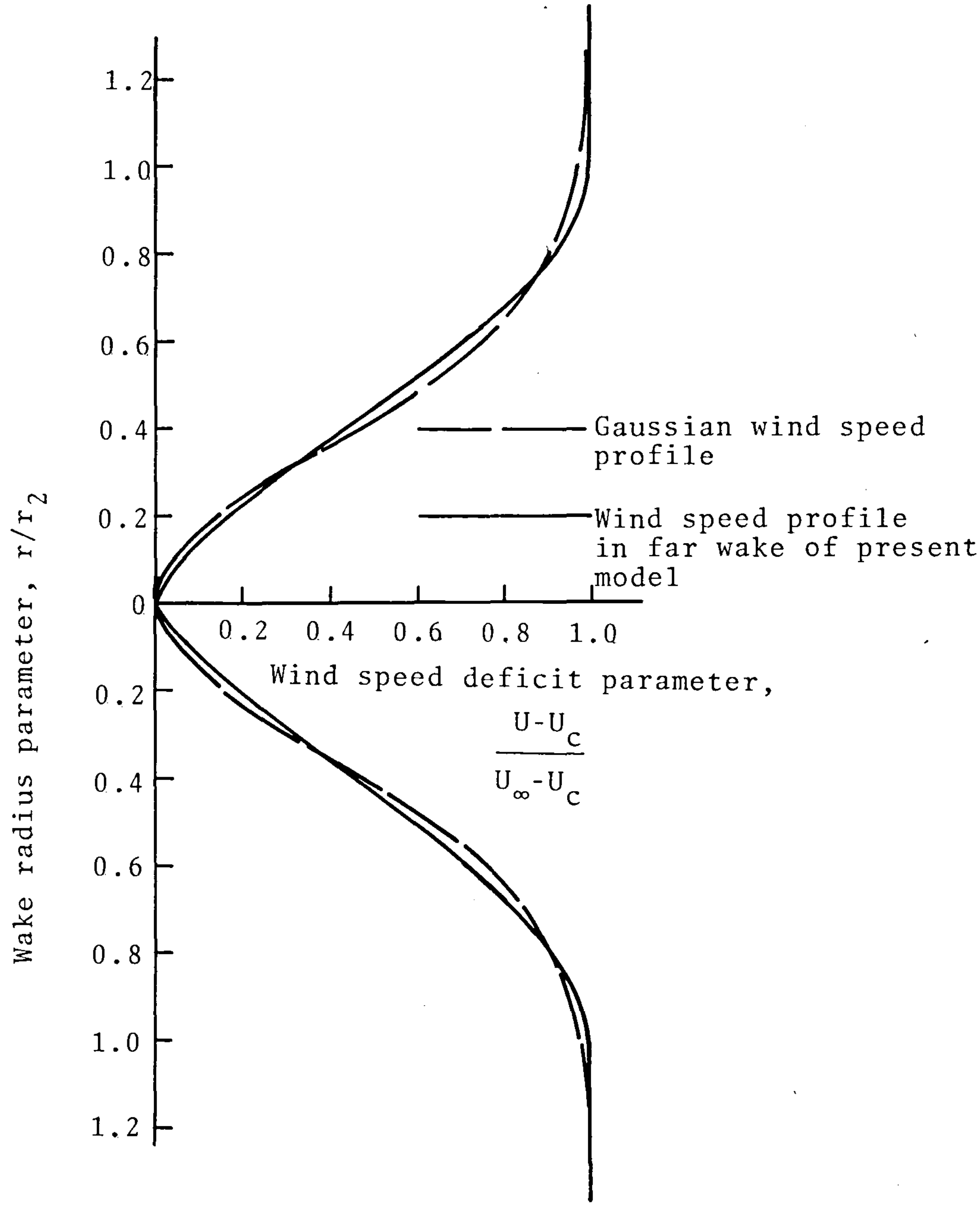
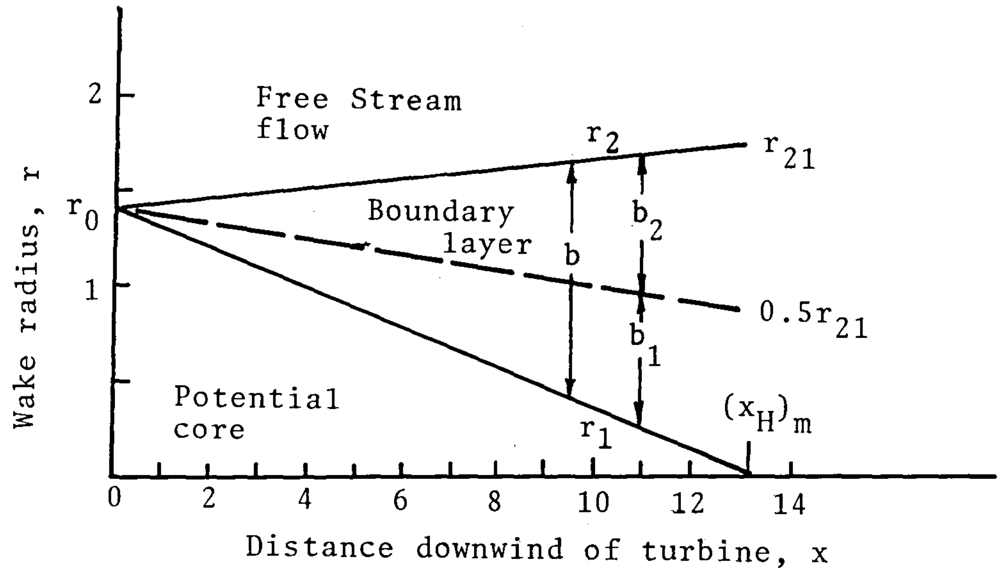
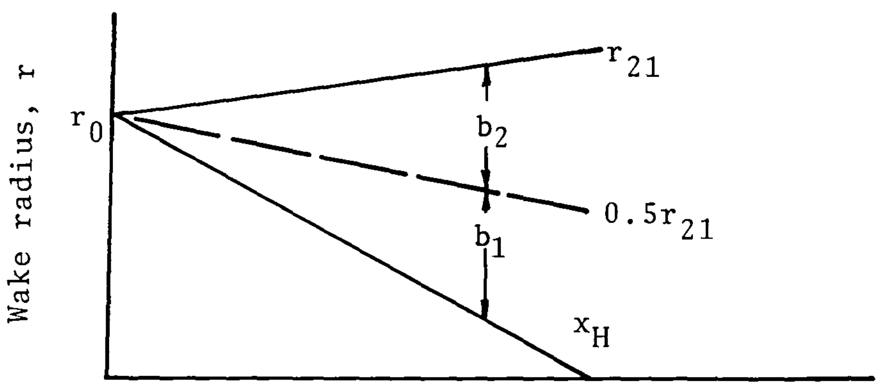


Figure 3-4. Comparison of the Gaussian wind speed profile used in plume dispersion analysis with the wind speed profile used in the far wake of the wake model.



(a) Region I for Abramovich solution (no ambient turbulence)



(b) Region I with ambient turbulence

Figure 3-5. - Geometry of Region I of the wake for calculating the downwind extent of Region I.

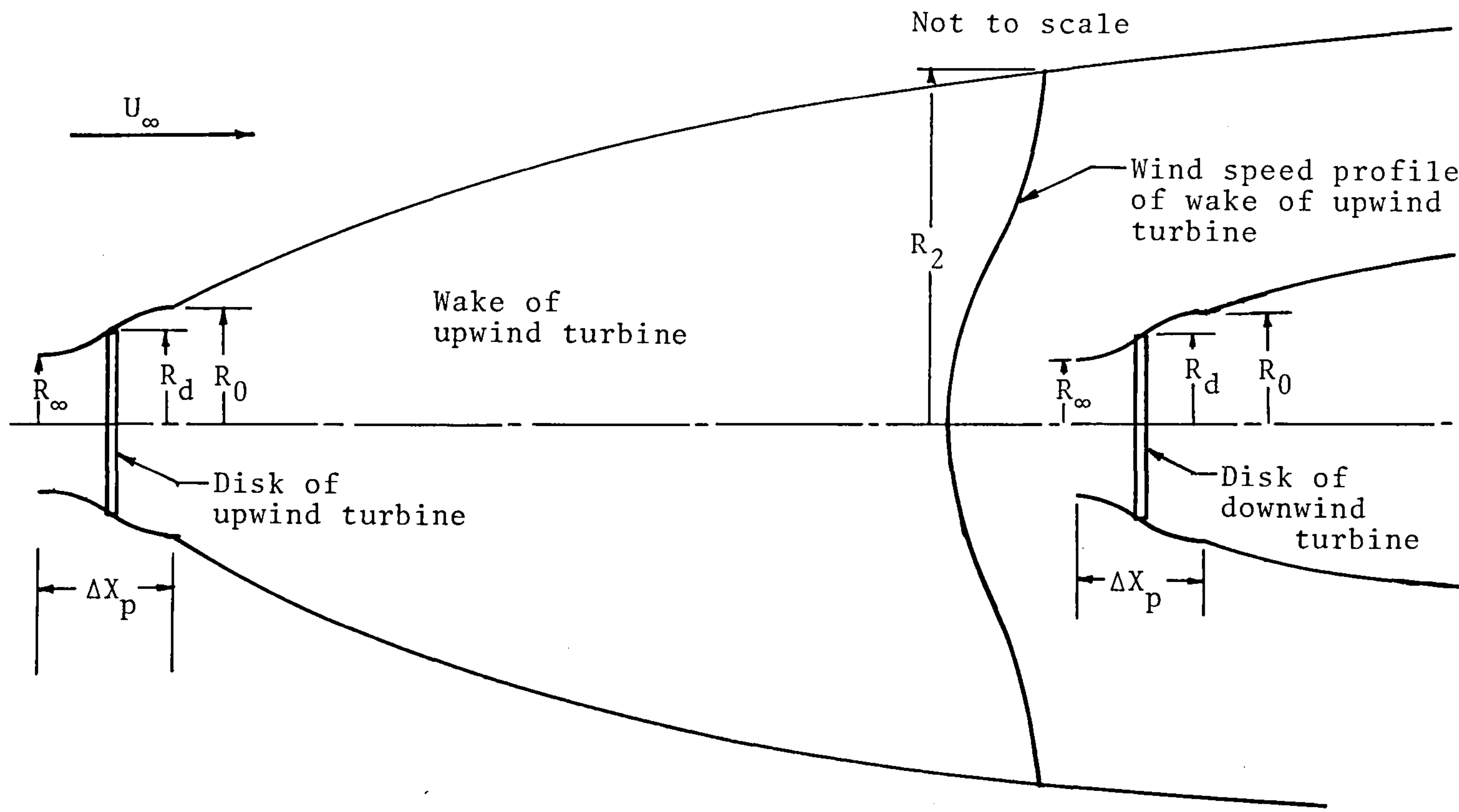


Figure 3-6. - Geometry of one turbine centered in the wake of another identical turbine.

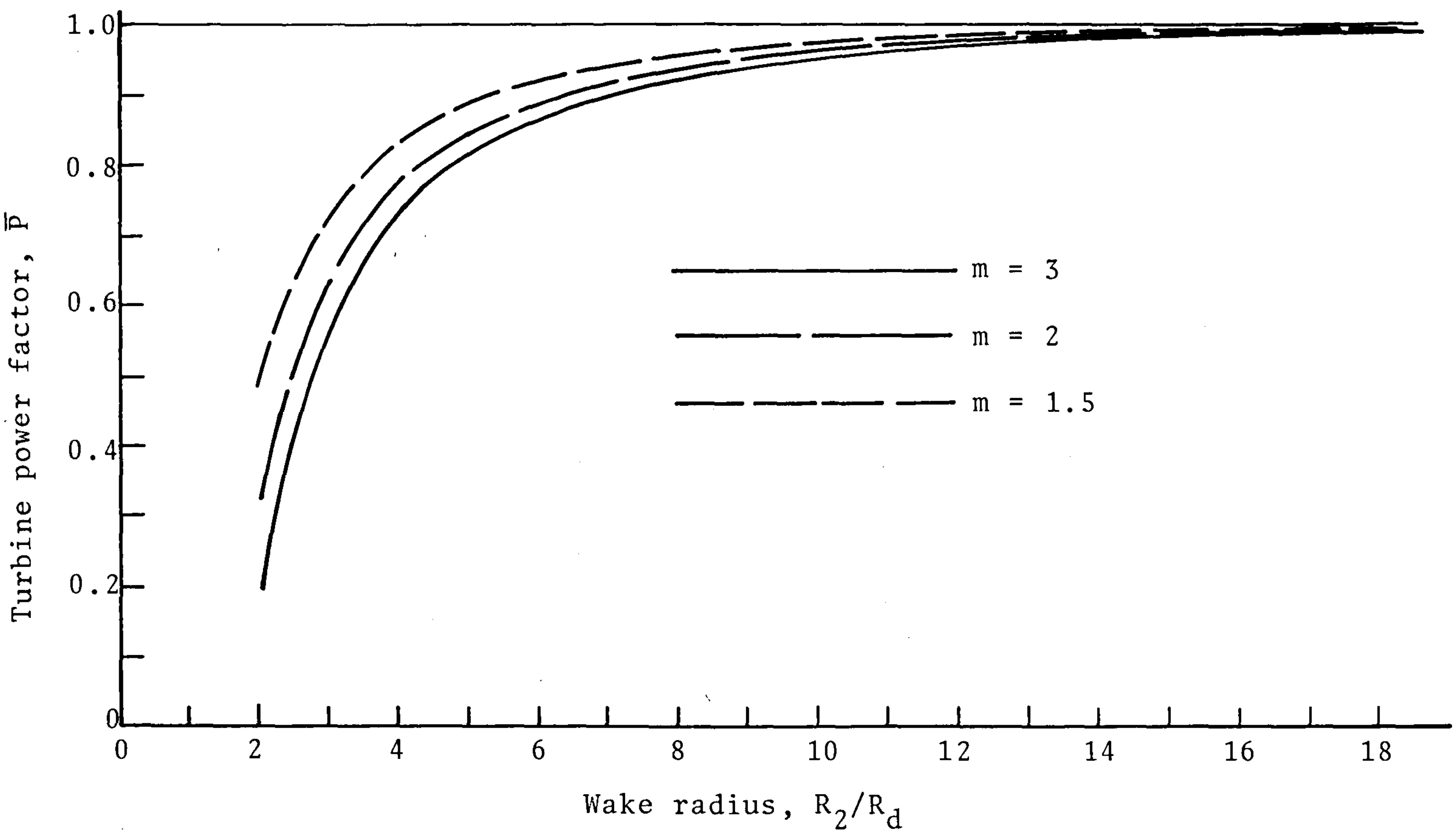


Figure 3-7. - Turbine power factor as a function of wake radius.

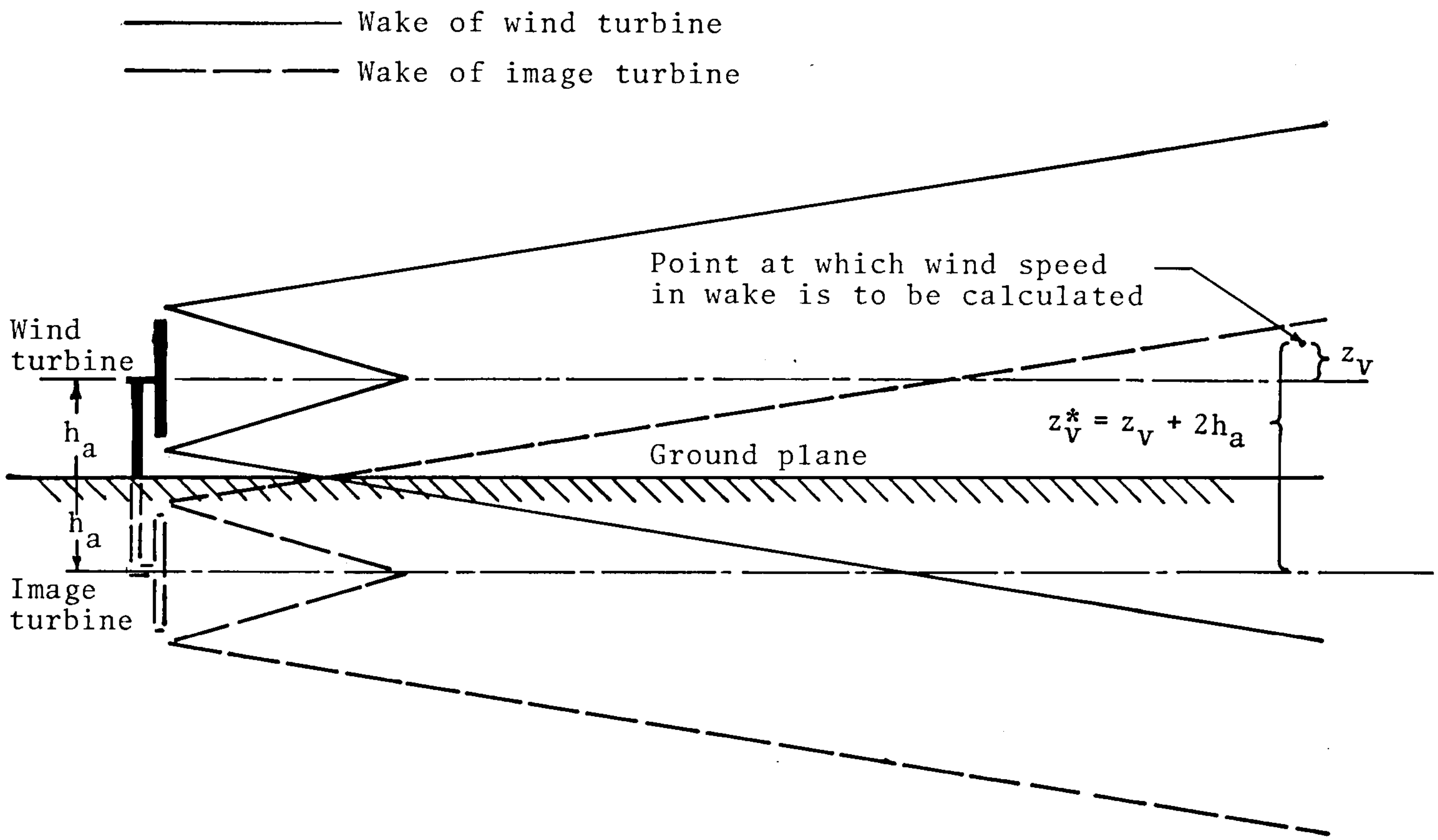


Figure 3-8. - Geometry for calculation of ground plane effect by image technique.

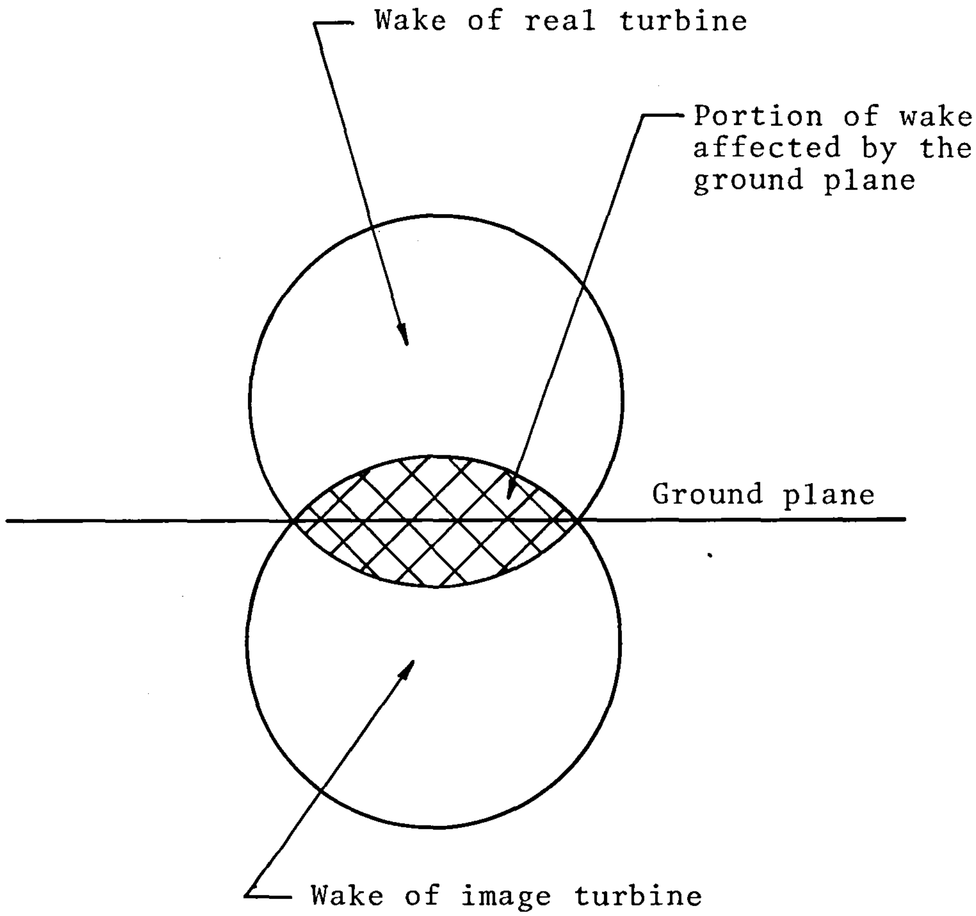


Figure 3-9. Illustration of the portion of the wake affected by the ground plane.

4.0 COMPARISON WITH PREVIOUS MODEL

There are some differences between the model presented in the previous section and the AeroVironment model presented in reference 8. For the purpose of completeness, those differences are documented in this section.

4.1 Wake Growth Rate Due to Ambient Turbulence

For the calculation of wake growth due to ambient turbulence, the AeroVironment model uses the expression

$$\frac{dR_2}{dX} \equiv - \frac{\alpha}{0.51} \tag{4-1}$$

where α is a direct data input for the calculation. Methods for determining the value of α were suggested, but not detailed. These included evaluating α from the effective turbulence intensity of the atmosphere at the site, or from the growth rates of smoke or polutant plumes as given in atmospheric dispersion theory. The present model uses the growth rate due to ambient turbulence as

$$\frac{dR_2}{dX} \equiv \alpha \tag{4-2}$$

where α is calculated internally from input considerations of Pasquill stability class designation for the atmosphere as given in dispersion theory. This is done in conjunction with the Taylor assumption for the relationship between velocity deficit profile and admixture profile (corresponds to the 1/0.51 in the AeroVironment formulation).

The AeroVironment model has provision for different values for vertical and lateral (normal to the wind direction) components of wake growth due to ambient turbulence. Under these conditions, the wake (out of ground effect) assumes an elliptical cross section instead of a circular cross section. This condition was not included in the present version for simplicity and because of the uncertainty about the appropriate procedure for relating unequal wake growth rates to an axisymmetric flow solution. In the present model, the wake growth rate due to ambient turbulence was taken as the lateral growth rate.

4.2 Support Tower

The effect of the wake of the turbine support tower was not included in the present version because it was believed to be an unwarranted complexity. Tower wake relationships depend on the type of construction (i.e., lattice or tubular) and the interaction between the tower wake and the turbine wake. Furtherfore, the effects of the tower wake should be diminished in the far wake of the turbine, which is the principal region of interest. Thus, the tower wake effect was deemed negligible compared to the other uncertainties in the model formulation.

4.3 Calculations for Region I

The calculation of r_{21}, the wake radius at the end of Region I was done differently in the revised model than in the AeroVironment model. The AeroVironment model used equation (5.21') of Abramovich. At the end of Region I, the boundary layer width, b, is r_{21}. In Abramovich's notation, y is the distance from the initial wake radius to the boundary of the potential core. Therefore, at the end of Region I, $y_1 = r_0$, and equation (5.21') of Abramovich can be written as

$$\frac{r_0}{r_{21}} = 0.416+0.134m+0.021\frac{r_{21}}{r_0}(1+0.8m-0.45m^2) \qquad (4\text{-}3)$$

This is a quadratic equation in r_{21}/r_0 which is solved for r_{21}/r_0 to give

$$\frac{r_{21}}{r_0} = \frac{-f+\sqrt{f^2+4g}}{2g} \qquad (4\text{-}4)$$

where

$$f = 0.416+0.134m \qquad (4\text{-}5)$$

$$g = 0.021(1+0.8m-0.45m^2) \qquad (4\text{-}6)$$

The positive sign was chosen for the solution to the quadratic equation so that r_{21}/r_0 is positive.

Although this approach is correct, it was deemed to be unnecessarily cumbersome. The parameter, r_{21}/r_0 is given directly by Abramovich, equation (5.19) as

$$\frac{r_0}{r_{21}} = \sqrt{0.214+0.144m} \tag{4-7}$$

This equation (same as eq. (3-34) was deemed to be simpler than that presented in the AeroVironment model. Numerically, the difference between these two methods is less than 0.4% for values of m between 1 and 3.

In the AeroVironment model, only one approach for calculating the downwind extent of Region I was presented. This was the first approach (r_1 approach) described in Appendix D. After consideration of this approach, it was realized that the second approach (r_2 approach) described in Appendix D was equally plausible from a physical phenomenological point of view, but the r_2 approach gave much different results. This paradox prompted the investigation of other approaches, and the seven approaches outlined in Appendix D resulted. The approach which was eventually chosen was not the approach originally presented in the AeroVironment model. The reader is referred to Appendix D for the description of the AeroVironment approach and the six other approaches considered.

4.4 Wake Growth in Region III

The AeroVironment model attempted to simplify the calculation of wake growth in Region III in order to circumvent the necessity of the numerical integration in Region III. The basic idea by which the calculation procedure was to be simplified was to perform the numerical integration externally to the wake program for different sets of values of α and m. The numerical integration was performed over an interval of x of $10r_0$ beginning with the wake radius r_{22}, which is a function of m only. Once the wake radius was known at the beginning and end of Region III, the effective ambient turbulence was defined as that ambient turbulence which alone would give the same wake growth in Region III as the combination of mechanical turbulence and ambient turbulence had given in Region III by the numerical integration. For example, for $m = 3$ and $\alpha = 0.1$,

$$r_0 = 1.414 \tag{4-8}$$

$$r_{22} = 1.988 \tag{4-9}$$

$$x_N = 5.488 \tag{4-10}$$

The downwind extent of Region III is $10r_0$ or at

$$x = x_N + 10r_0 = 19.63 \tag{4-11}$$

From the numerical integration of equations (3-57) through (3-62), $r_2 = 3.43$ at $x = 18.42$, and $r_2 = 3.57$ at $x = 19.83$. By linear interpolation, $r_2 = 3.55$ at $x = 19.63$. Therefore, in Region III

$$\alpha_e = \left(\frac{dr_2}{dx}\right)_{avg} = \frac{3.55-1.99}{19.83-5.49} = 0.1088 \tag{4-12}$$

Therefore, for $m = 3$ and $\alpha = 0.1$, the effective growth rate in Region III is $\alpha_e = 0.1088$.

In the AeroVironment model, the numerical ingetration was conducted external to the main wake computer program. Plots of α_e as a function of α were generated for several selected values of m, and α_e was an input paramter for the wake computer program. In Region III, the wake growth rate was assumed to be constant at a value of α_e. In Region IV (downwind of Region III), the wake growth rate was assumed to be constant at a value of α. That is, downwind of $x = x_N+10r_0$, the wake growth was assumed to be due to ambient turbulence alone with no contribution to wake growth due to mechanical turbulence. In this discussion, the definition of α used in this report is the growth rate of the wake radius due to ambient turbulence. In the AeroVironment report, α was defined slightly differently, and the growth rate of the wake radius due to ambient turbulence is shown in the AeroVironment report as $\alpha/0.51$.

The method used in the AeroVironment report is reasonably accurate, acsept for low values of ambient turbulence. At low values of ambient turbulence ($\alpha<0.1$), there is still a significant value of wake growth due to mechanical turbulence downwind of $x = x_N+10r_0$. Also, it was deemed appropriate that the model should assume the exact form of the Abramovich solution for $\alpha = 0$. The AeroVironment model does not do that since the wake growth rate in Region IV goes to zero for $\alpha = 0$. For these reasons, in the revised model, the numerical integration of equations (3-57) through (3-62) was made an integral part of the model.

5.0 SAMPLE PLOTS

This section contains sample plots made with the turbine wake computer program. The plots were made from seventeen runs of the program. Illustrated in this section are the input of parameters for the computer program and the output obtained. The first part of this section presents complete sets of plots from the computer program. These figures include plots of the Abramovich solutions (i.e., no ambient turbulence). The second part of this section shows the effect of changes in wake variables on wake recovery. Results are presented for changes in ambient turbulence factor; power law exponent, height of the axis of the turbine rotor, and initial velocity ratio.

5.1 Wake Plots

Abramovich solutions. - Initially, it was desired to obtain plots for the Abramovich solution for the coflowing jet (α = 0). The input parameters are shown below.

VALUES OF INPUT PARAMETERS FOR TURBINE WAKE PLOTS FOR ABRAMOVICH SOLUTION

ST = 0.0	J = 5
CPM = 1.5	IO = 1
AH = 50.	NP = 0
VHO = 1.0	XNPT = 5., 10., 20., 35., 50.
WEXP = 0.	ZO = 48.5
RRR = 1.0	DZZ = 0.5
	DYY = 0.05

The input value, ST = 0.0, makes the ambient turbulence zero. The value of CPM = 1.5 was chosen as a typical value for large wind turbines. The value AH = 50. was used to make the wake isolated from the ground effect. VHO = 1. was used to make all wind speed plots normalized by the free stream wind speed. The power law exponent, WEXP = 0., gives no variation in free stream wind speed with altitude. The rotor radius, RRR = 1.0 gives all geometric output in rotor radii. J = 5 indicates five downwind locations at which wind speed profiles are to be calculated. IO = 1 indicates that geomteric input is in rotor radii. NP = 0 indicates that the wind speed profiles are to be normalized by the free stream wind speed at the altitude of the profile. The values of downwind location at which wind speed profiles are to be generated are defined by XNPT(j) = 5., 10., 20., 35., and 50.. For the wind speed profiles, Z0 = 48.5, and DZZ = 0.5. The altitudes at which the wind speed profiles are to be calculated are

$$z_i = z_0 + i\Delta z \qquad i = 0, \ldots, 7 \qquad (5\text{-}1)$$

Therefore, the altitudes of the lateral wind speed profiles are 48.5, 49.0, 49.5, 50.0, 50.5, 51.0, 51.5, and 52.0. Since $h_a = 50.$ for the input values of z_0 and Δz, the fourth wind speed profile is at the hub altitude. The second wind speed profile is at the lower edge of the turbine disk, and the sixth wind speed profile is at the upper edge of the turbine disk. The value, DYY = 0.05, indicates that calculations are made at intervals of 0.05 rotor radii.

Figure 5-1 shows the set of computer-generated plots for the set of input data shown in the preceding table. The first plot shows the wake radius and the wake width as a function of the downwind coordinate, x. Since the wake is not near the ground, the wake height above the ground is not shown. The first plot also lists the important wake parameters for the set of plots. The second plot shows Δu_c as a function of x, and the third plot shows U_c/U_∞ as a function of x. The fourth plot shows the variation of turbine power factor. Since the turbine power factor is undefined in Regions I and II, the plot is shown downwind of x_N only. The fifth through the ninth plots show the lateral wind speed profiles for the downwind locations shown in the table. The altitude of each profile is shown in rotor radii relative to the turbine hub. Since the wake is far from the ground, the vertical wind speed profile at $y = 0$ is symmetrical to the lateral profile at $Z_v = R_d$. It was, therefore, not shown.

Figure 5-2 shows a set of similar plots for $m = 2.0$, and figure 5-3 shows the values for $m = 3.0$. The first plots of each of figures 5-1, 5-2, and 5-3 is similar to the plots given by Abramovich in figure 5.17 of reference 2. The second plot in each of these figures is similar to figure 5.20 in reference 2. In the Abramovich plots, the geometric parameters are normalized by the initial wake radius, R_0. In figures 5-1, 5-2, and 5-3, the geometric parameters are normalized by the turbine dish radius, R_d. The relationship between R_0 and R_d is given by equation (3-18).

Wakes in ground effect. - The input parameters for the wake in ground effect are listed below. The hub of the turbine is 1.35 rotor radii above the ground. This is typical of very large wind turbines. The first lateral wind speed profile (i = 0; z_0 = 0.35) is at the lower edge of the turbine disk, and the third lateral wind speed profile (i = 2) is at the rotor hub.

VALUES OF INPUT PARAMETERS FOR TURBINE WAKE PLOTS
FOR WAKE IN GROUND EFFECT

ST = 0.0	J = 5
CPM = 1.5	IO = 1
AH = 1.35	NP = 0
VH0 = 1.0	XNPT(j) = 5., 10., 20., 35., 50.
WEXP = 0.15	Z0 = 0.35
RRR = 1.0	DZZ = 0.5
	DYY = 0.05

Figure 5-4 shows the set of wake plots for the input data shown above. Since the wake is near the ground, the height of the top of the wake above the ground is also shown in the first plot, and a vertical wind speed profile plot is included. In the plot of the vertical wind speed profiles, the solid line going across the plot is the wake center. The short horizontal lines at the left of the plot are the bottom and top of the turbine disk.

In the plot of the normalized wake boundaries (Fig. 5-4(1)), the point at which the wake radius equals h_a, the height of the rotor hub above the ground, is the point at which the wake intersects the ground plane. Initially, the effect of the ground plane affects only the lower part of the wake, but it can spread to affect the wake center, although the magnitude of the effect at the wake center is negligibly small because of the large wake radius (and correspondingly small wid speed deficit) necessary for the effect of the ground plane to reach the wake center. It is recalled from the development of the turbine power factor that the turbine power factor does not include the effect ot the ground plane.

The effect of the ground on the wind speed profiles is seen by comparing the wind speed profiles for x = 50 from Figures 5-1 and 5-4. In Figure 5-1, the profiles for altitudes of -1.0 and 1.0 rotor radii from the rotor hub are identical because of the symmetry of the wake. However, because of the ground effect, the wind speed profiles at altitudes of -1.0 and 1.0 rotor radii relative to the hub are not identical in Figure 5-4. The lower altitude exhibits a lower wind speed in the wake because of the ground effect. At x = 50, it is only the lowest altitude which has been affected by the ground. As the wake expands further, the effect of the ground affects the wind speed profiles of higher altitudes. This is seen in later plots where ambient turbulence causes faster expansion of the wake.

For all of the previous figures, there is no ambient turbulence. Figure 5-5 shows a plot set for ambient turbulence represented by Pasquill stability class E. The data input is identical with that shown in the table, except that ST = -5. for Pasquill stability class E. From Table 3-4 and equation (3-28), the wake growth rate due to ambient turbulence is $\alpha = 0.099$.

Figure 5-6 shows a plot set for Pasquill stability class D (ST = -4.). The wake growth rate due to ambient turbulence is $\alpha = 0.136$. Figure 5-7 shows a plot set for Pasquill stability class C (ST = -3.). The wake growth rate due to ambient turbulence is $\alpha = 0.205$.

5.2 Effect of Principal Wake Parameters

In the following discussion, the effect of the principal wake parameters upon the wake is investigated. The parameters which are varied are the ambient turbulence as represented by the Pasquill stability class; the initial velocity ratio, m; the height of the rotor hub, h_a; and the power law coefficient of the free stream wind speed profile, γ.

Effect of ambient turbulence. - Figure 5-8 shows the effect of ambient turbulence on the wake radius. Figure 5-8 is a composite of the wake radius as shown in the first plot of Figures 5-4 through 5-7. Figure 5-9 shows the effect of ambient turbulence of Δu_c. Figure 5-10 shows the effect of ambient turbulence on U_c/U_∞. Figure 5-11 shows the effect of ambient turbulence on the turbine power factor. It is seen that ambient turbulence is a very important factor in the recovery of wakes of large wind turbines. From Table 3-3, it is recalled that most turbine operation occurs with Pasquill stability class D, and Pasquill class D represents the ambient turbulence level to which inter-turbine spacing should be designed.

The turbine power factor is the most significant indicator of wake recovery. From Figure 5-11, for Pasquill stability class D, the turbine power factor is approximately 0.9 at a distance of 35 rotor radii downwind of the turbine. While the ambient turbulence has a significant impact on the growth in the wake radius, its impact on the turbine power factor is not great. For comparison with the 0.90 power factor for Pasquill stability class D, the turbine power factor is 0.85 for Pasquill stability class E and 0.95 for Pasquill stability class C at 35 rotor radii downwind of the turbine.

It is clear that a point of diminishing returns is reached in the re-energization of the turbine wake. From Figure 3-7, for m = 1.5, a turbine power factor of 0.9 is reached at $r_2 \cong 5.4$ Figures 5-8 and 5-11 confirm this number since Figure 5-11 shows that $\overline{P} = 0.9$ at approximately x = 34 for Pasquill stability class D, and Figure 5-8 shows that $r_2 = 5.4$ at x = 34 for Pasquill stability class D. From Figure 3-7, the turbine power factor reaches a value of 0.95 at wake radius, $r_2 \cong 8$. From extrapolation of the wake radius for Pasquill stability class D in Figure 5-8, a wake radius of 8 occurs at $x \cong 56$. Thus, a distance of 35 wake radii is required for recovery from the initial wake to a turbine power factor of 0.9, and an additional distance of 21 rotor radii is required for recovery from 0.90 to 0.95 for the turbine power factor. Certainly, power recovery is very much slower after the turbine power factor reaches 0.9 than it is before it reaches 0.9.

For the far wake, Figure 3-7 is essentially a plot of turbine power factor as a function of downwind distance since the wake radius is a straight line function of the downwind coordinate, x. In the far wake, the wake growth rate is almost entirely due to ambient turbulence with very little contribution from mechanical turbulence. For Pasquill stability class D, a scaling factor of 1/0.136 should be applied to the wake radius axis of Figure 3-7 to convert it to distance downwind of the turbine, since the wake growth rate due to ambient turbulence is 0.136 for Pasquill stability class D. Interpreting Figure 3-7 in this way shows that a very large downwind distance is required for recovery of the last few percentage points for the turbine power factor. In practice, recovery to very high values of turbine power factor may be impractical because of the large additional distances required.

Effect of initial velocity ratio. - Figure 5-12 shows the effect of the initial velocity ratio, m, upon the wake radius. Figure 5-13 shows the effect of m upon Δu_C. Figure 5-14 shows the effect of m upon U_C/U_∞, and Figure 5-15 shows the effect of m upon the turbine power factor.

The initial velocity ratio has almost no effect upon the wake radius. The downwind extent of Region II, X_N, is almost totally unaffected by m. This is shown in Figures 5-13, 5-14, and 5-15. Figure 5-13 shows the relative wind speed deficit (i.e., the wind speed deficit at the wake center normalized by the wind speed deficit at the center of the initial wake). This relative wind speed deficit is not a strong function of m. However, because the wind speed deficit of the initial wake is much larger for a large value of m (by definition of m), the normalized wind speed at the wake center (as shown in Figure 5-14) strongly depends on m. Correspondingely, the turbine power factor shown in Figure 5-15 is a strong function of m. For the larger values of m, more power is extracted by each wind turbine (cf. equations (3-9) and 3-11)), but the inter-turbine recovery of the wake requires more space. The lines shown in Figure 5-15 approximate the cube of the lines shown in Figure 5-14.

<u>Effect of turbine rotor height</u>. - The wind speed profile plots are the only plots which show the effect of the ground on the wake. The ground effect can be seen best from the vertical wind speed profiles with $\gamma = 0$ (i.e., no variation in the free stream wind speed with altitude). Figures 5-16, 5-17, and 5-18 show the vertical wind speed profiles for values of h_a of 1.2, 1.35, and 1.65, respectively.

In the near wake (x = 5 and x = 10), the wake is approximately symmetric for $h_a = 1.35$ and $h_a = 1.65$. For the far wake (x = 20, x = 35, and x = 50) the wake is clearly not symmetric for all values of h_a. The effect of the ground upon the wind speed profiles can be clearly seen by comparing the profiles for x = 20 from the three plots. The retarding effect of the ground is clearly evident. For comparison, Figures 5-19, 5-20, and 5-21 show the same data as Figures 5-16, 5-17, and 5-18, but with a power law coefficient, γ, of 0.15.

<u>Effect of the power law coefficient</u>. - The power law exponent, γ, only affects the vertical wind speed profile. It can be seen on the vertical wind speed profile and on the lateral wind speed profile (if NP = 1 so that the wind speed in the wake is normalized by the free stream wind speed at the hub altitude). The effect of γ on the vertical wind speed profile may be seen by comparing Figures 5-13, 5-20, and 5-22. The value, $\gamma = 0.25$ in Figure 5-22 represents a very turbulent lower atmosphere.

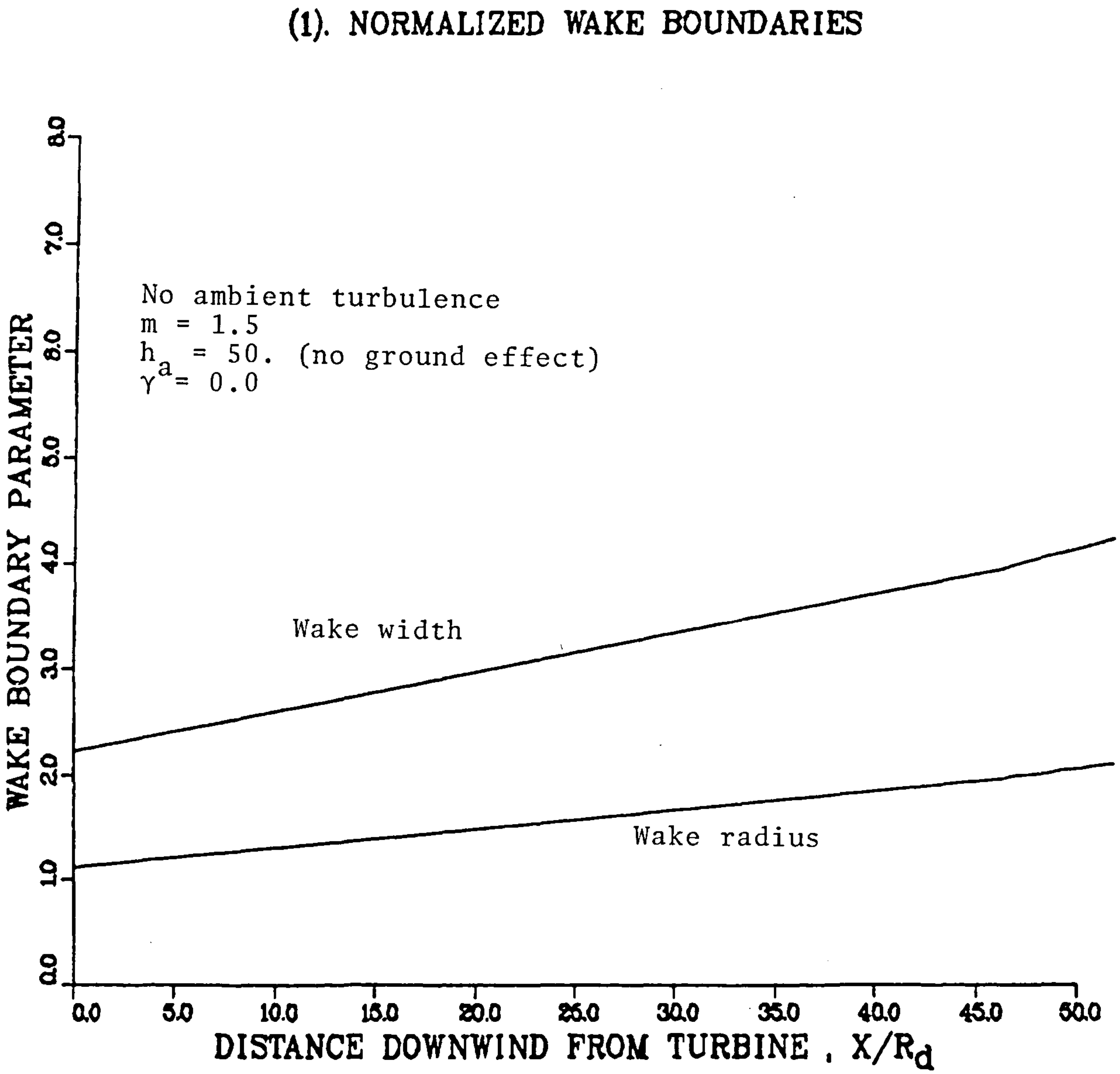


Figure 5-1. Wake plots for Abramovich solution for m = 1.5.

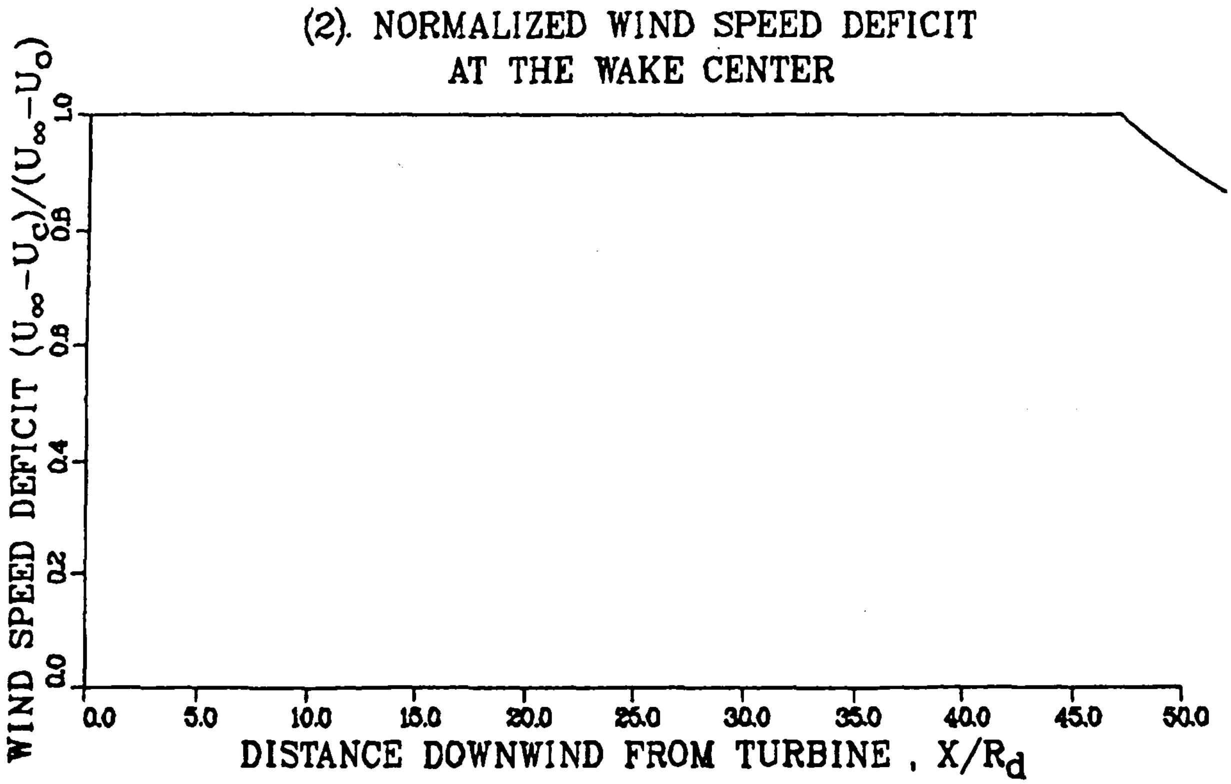


Figure 5-1. Wake plots for Abramovich solution for m = 1.5 (continued).

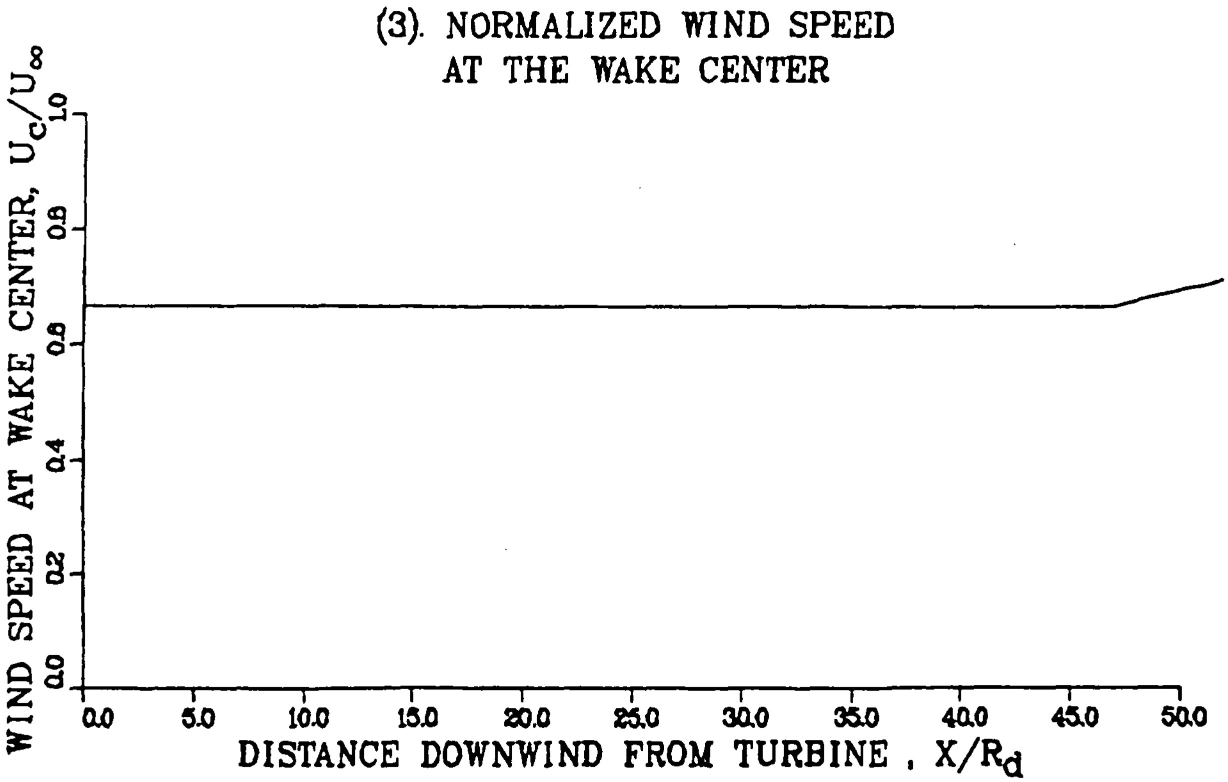


Figure 5-1. Wake plots for Abramovich solution for m = 1.5 (continued).

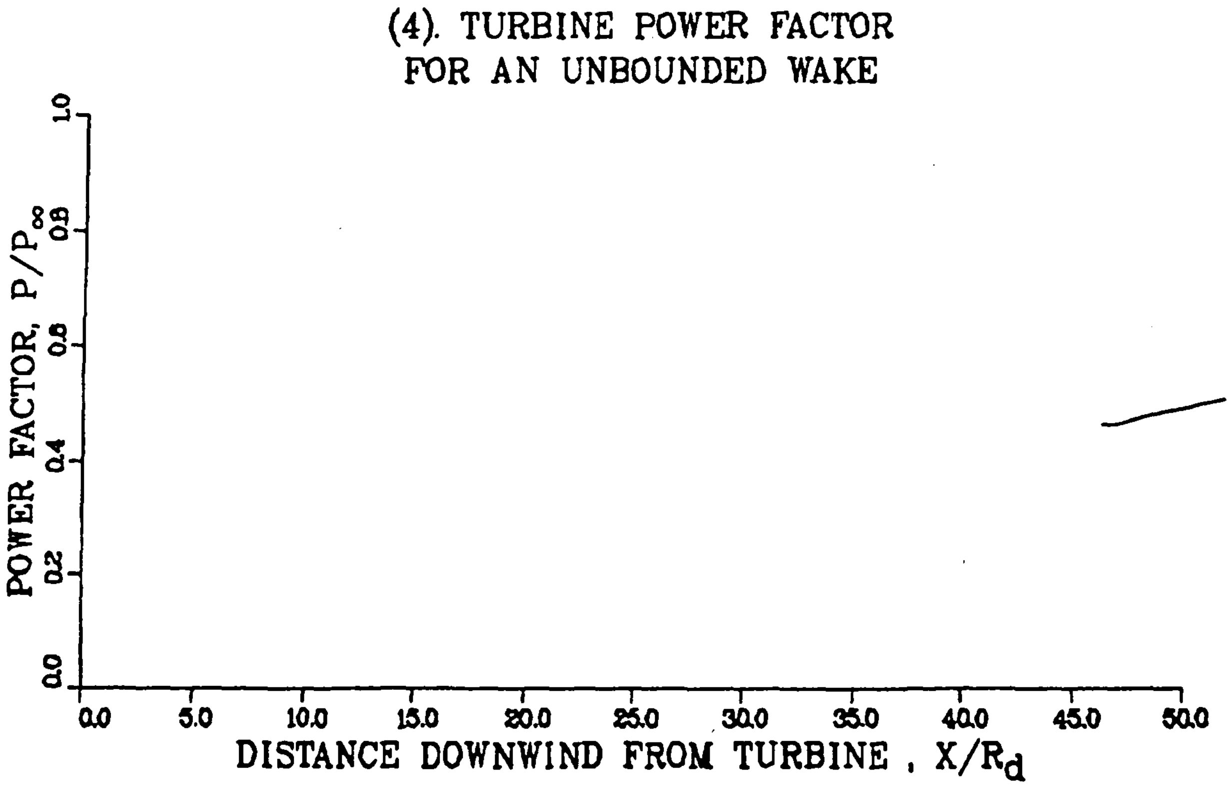


Figure 5-1. Wake plots for Abramovich solution for m = 1.5 (continued).

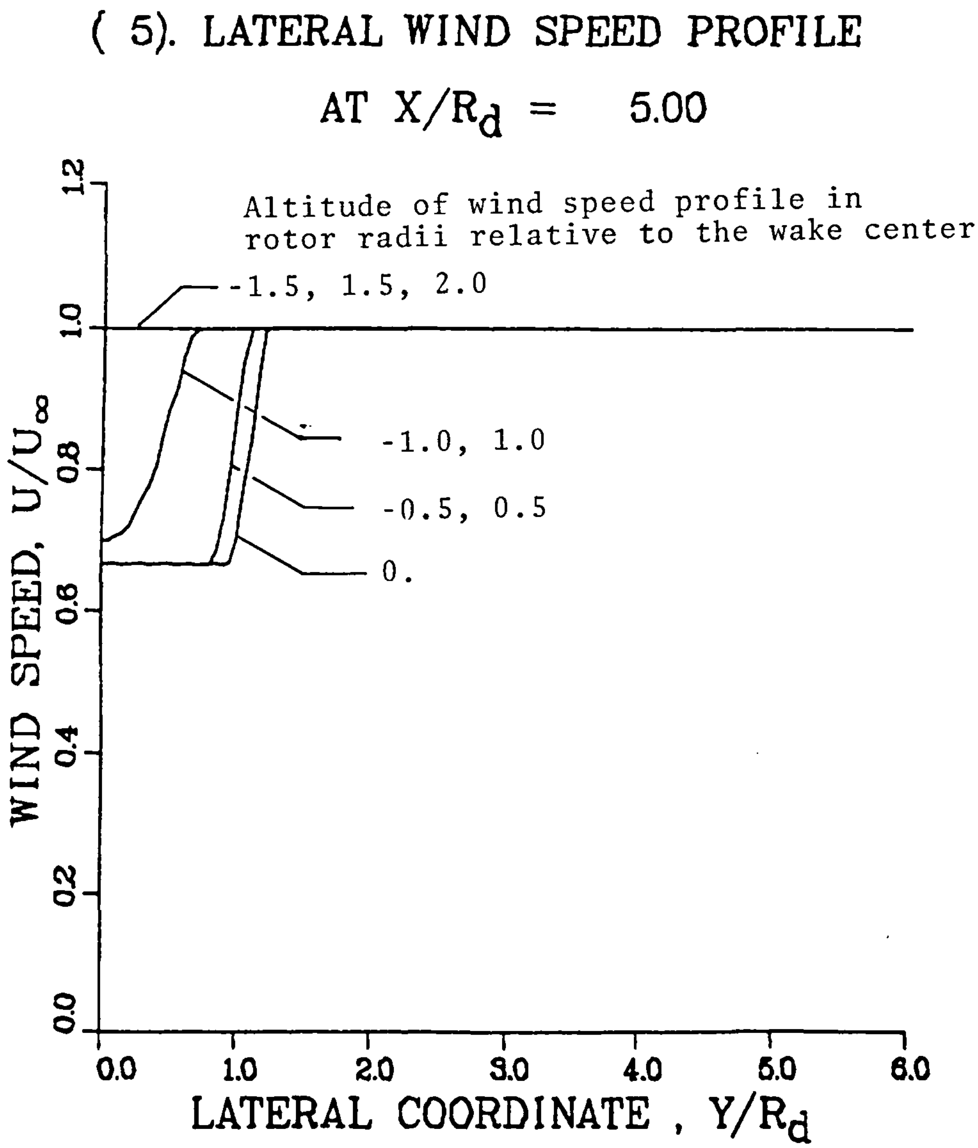


Figure 5-1. Wake plots for Abramovich solution for m = 1.5 (continued).

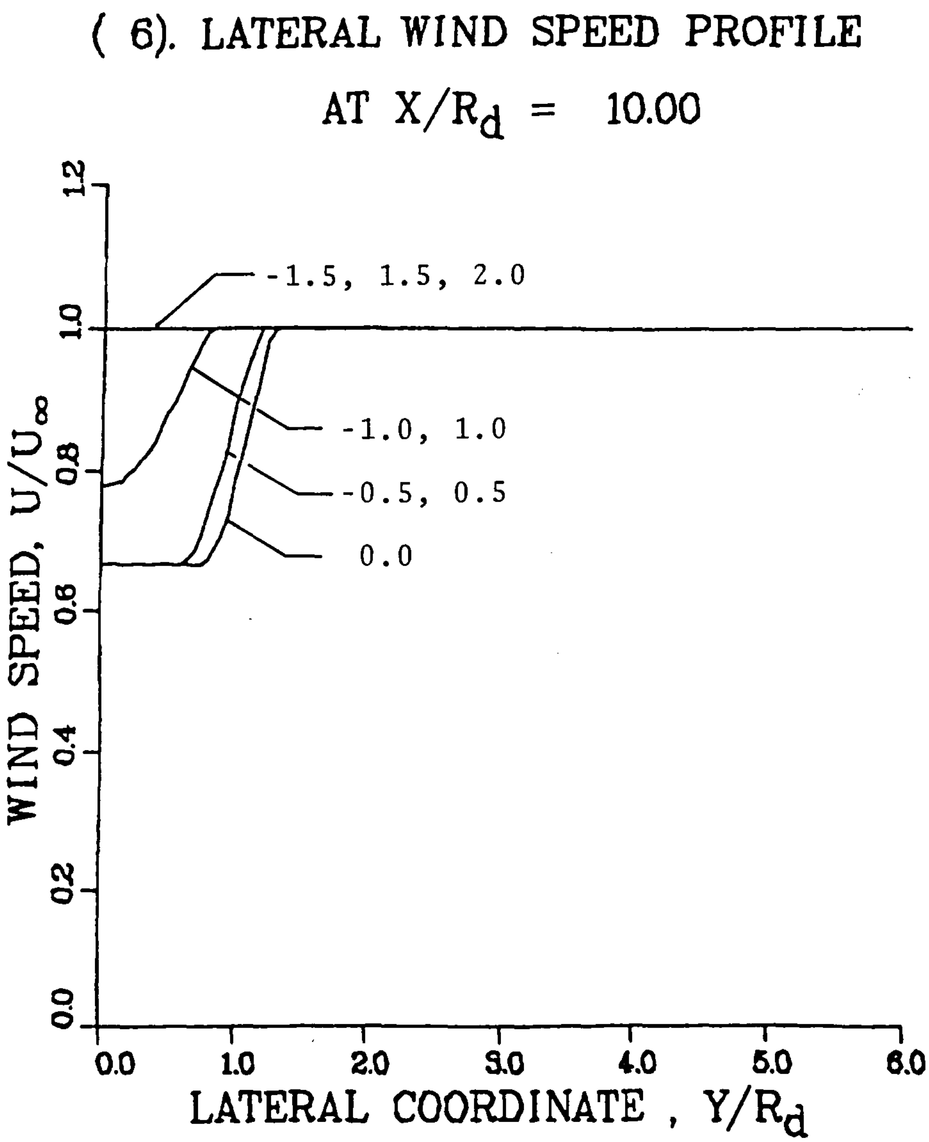


Figure 5-1. Wake plots for Abramovich solution for m = 1.5 (continued).

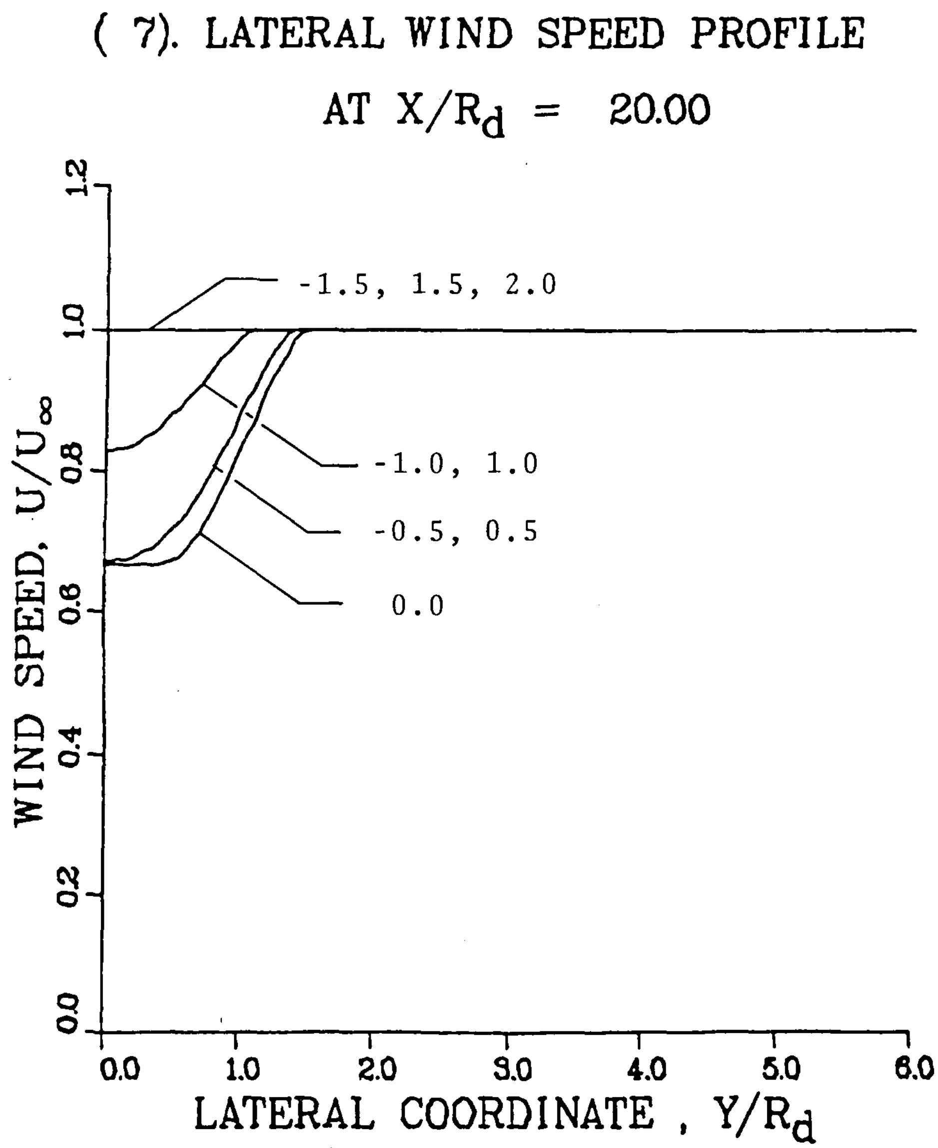


Figure 5-1. Wake plots for Abramovich solution for $m = 1.5$ (continued).

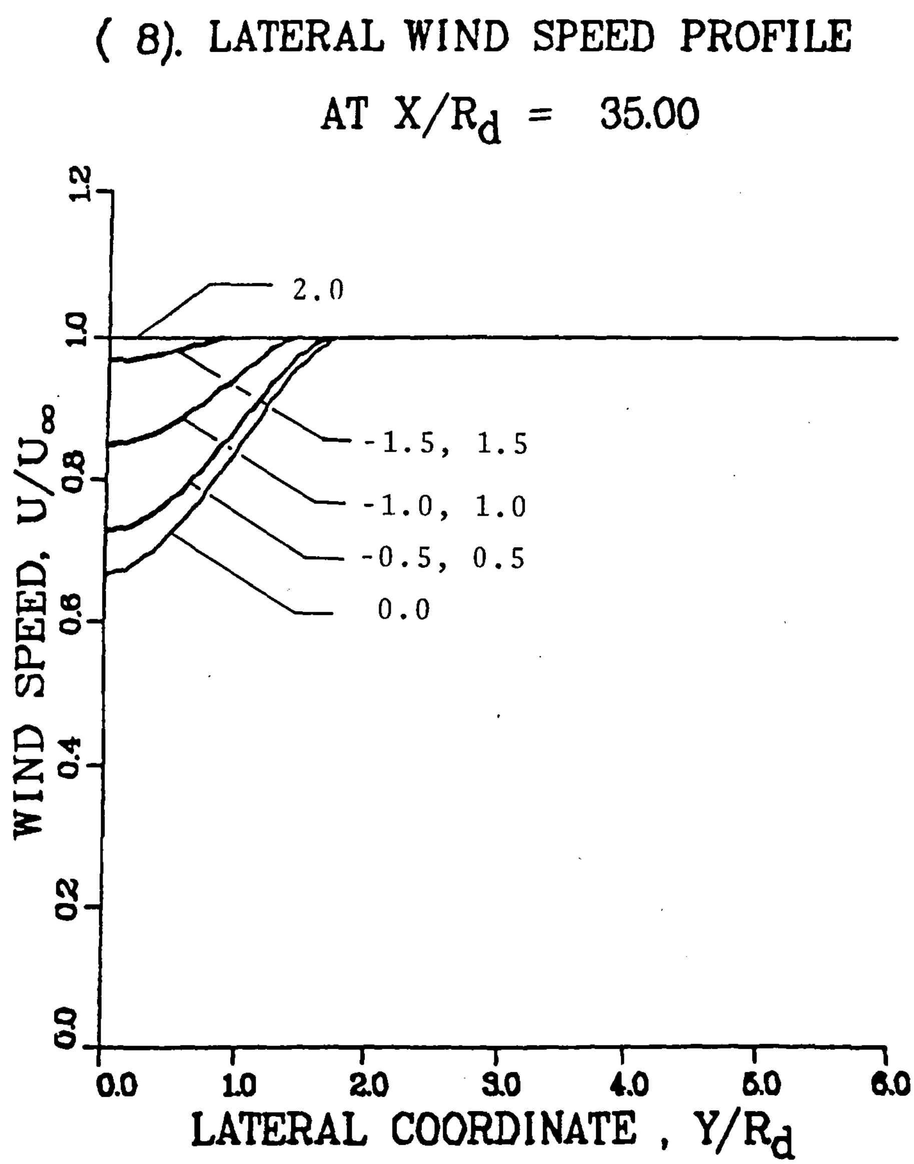


Figure 5-1. Wake plots for Abramovich solution for m = 1.5 (continued).

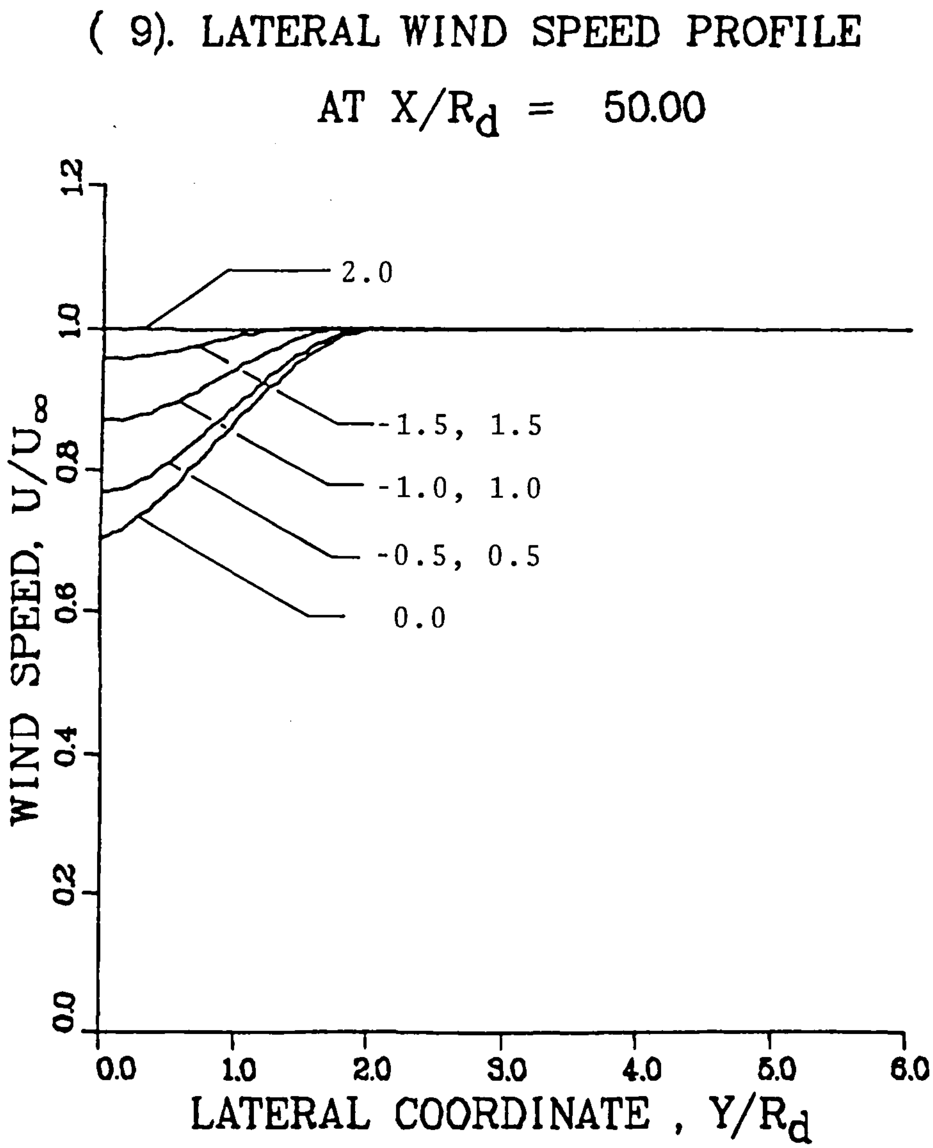


Figure 5-1. Wake plots for Abramovich solution for m = 1.5 (concluded).

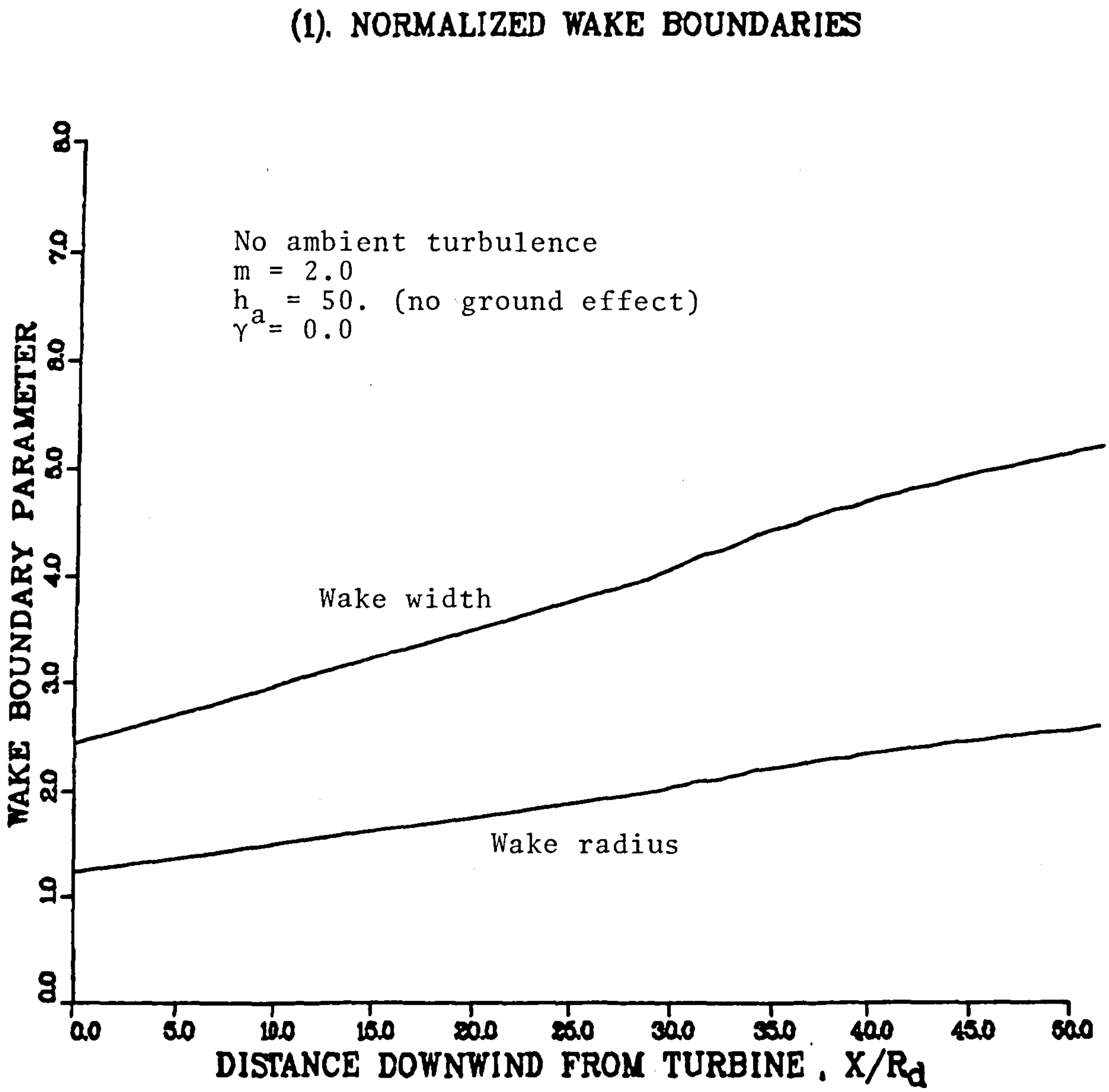


Figure 5-2. Wake plots for Abramovich solution for m = 2.0.

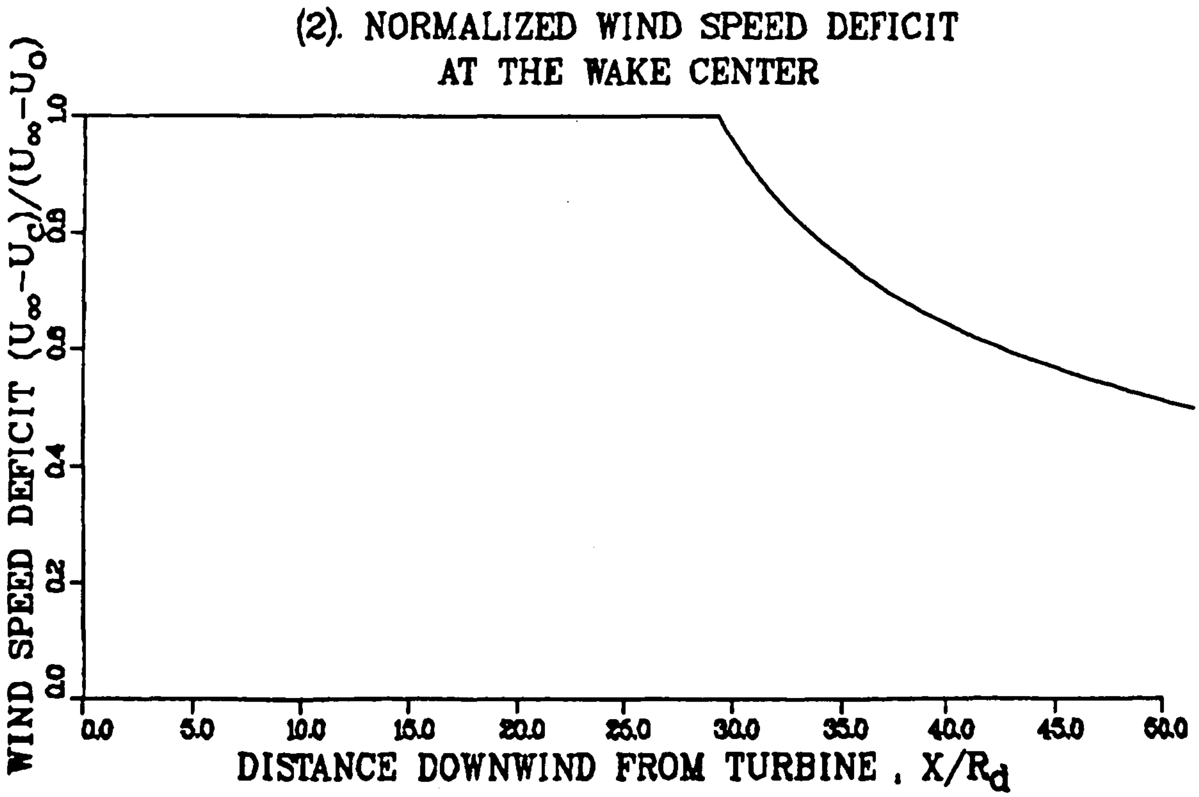


Figure 5-2. Wake plots for Abramovich solution for m = 2.0 (continued).

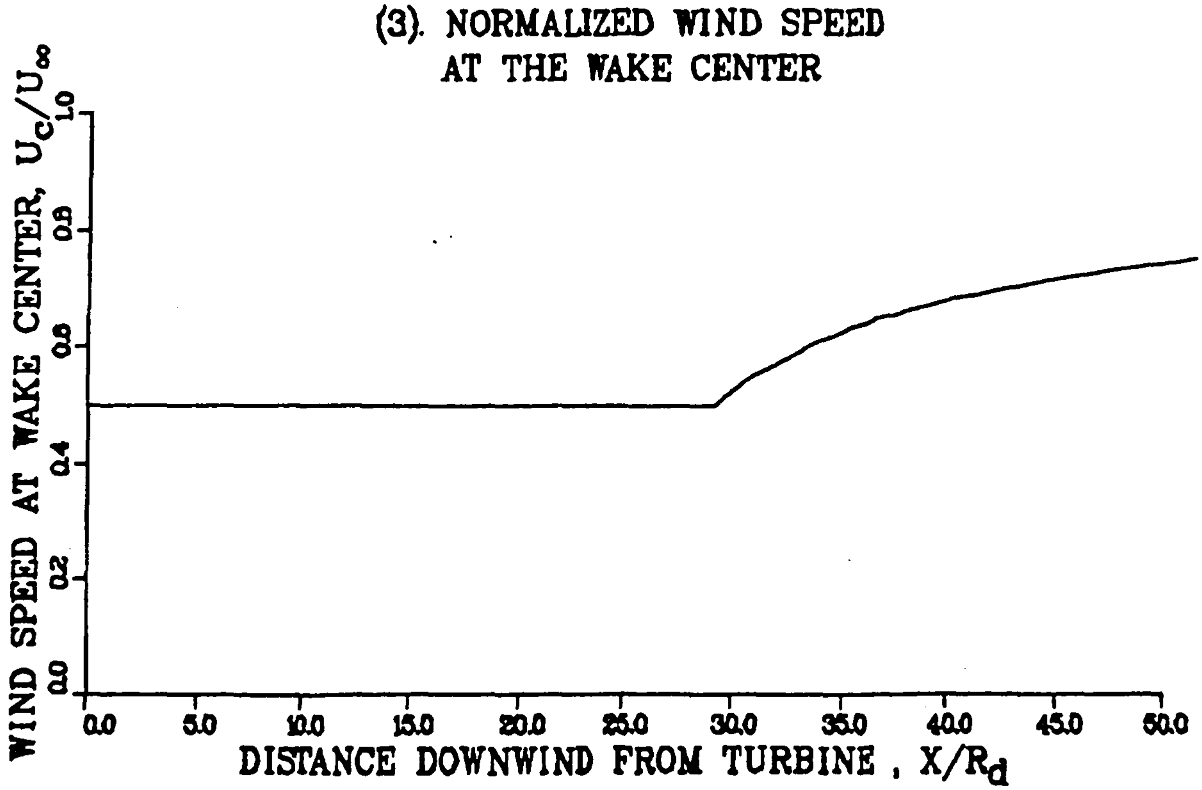


Figure 5-2. Wake plots for Abramovich solution for m = 2.0 (continued).

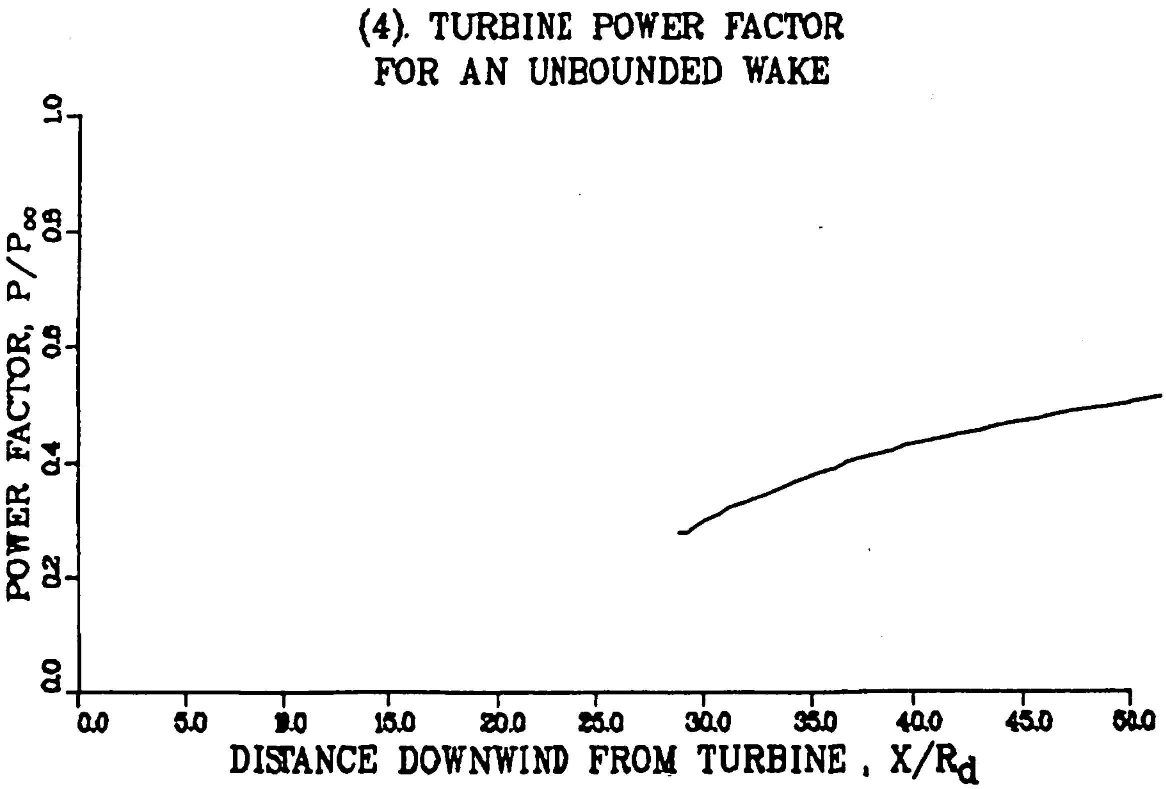


Figure 5-2. Wake plots for Abramovich solution for m = 2.0 (continued).

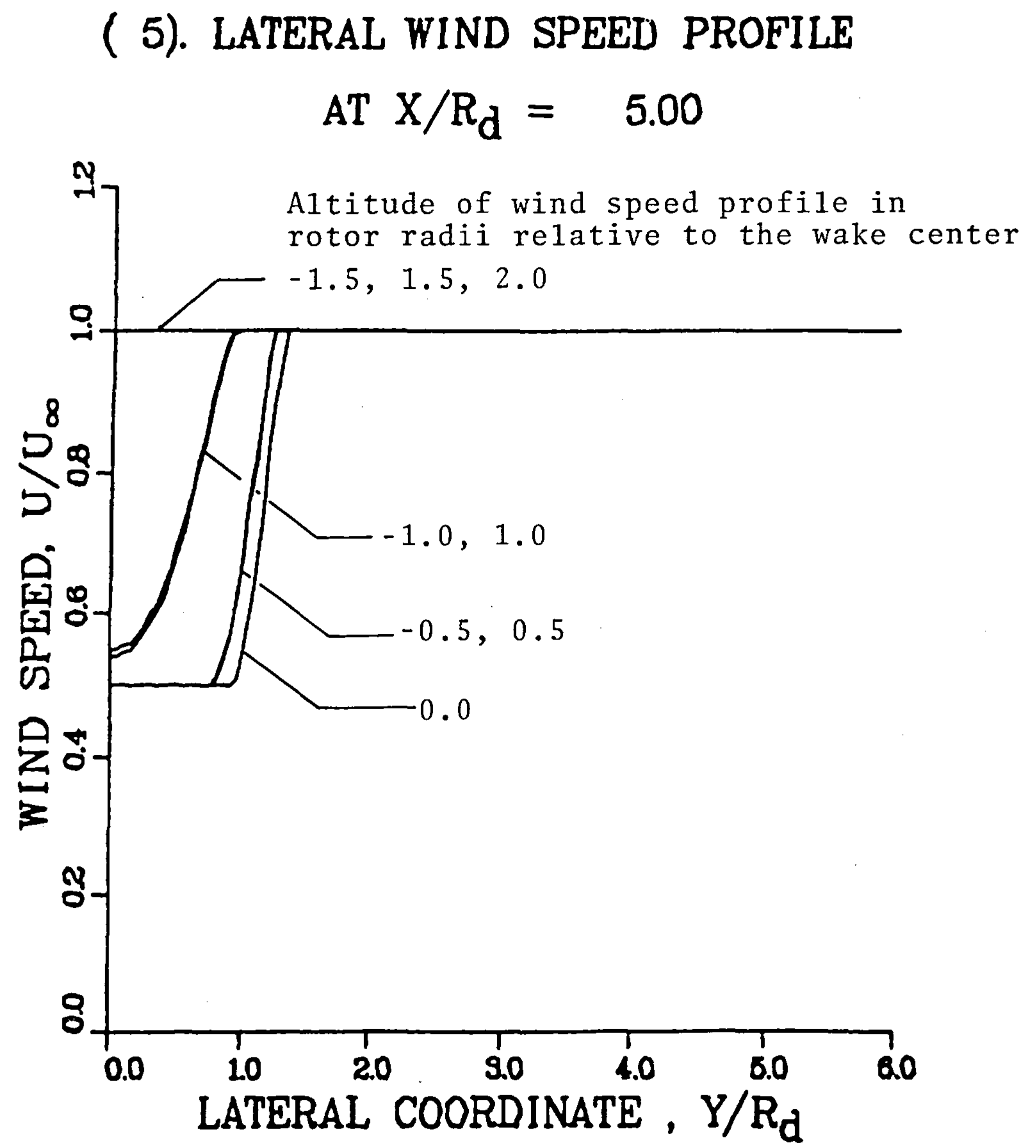


Figure 5-2. Wake plots for Abramovich solution for m = 2.0 (continued).

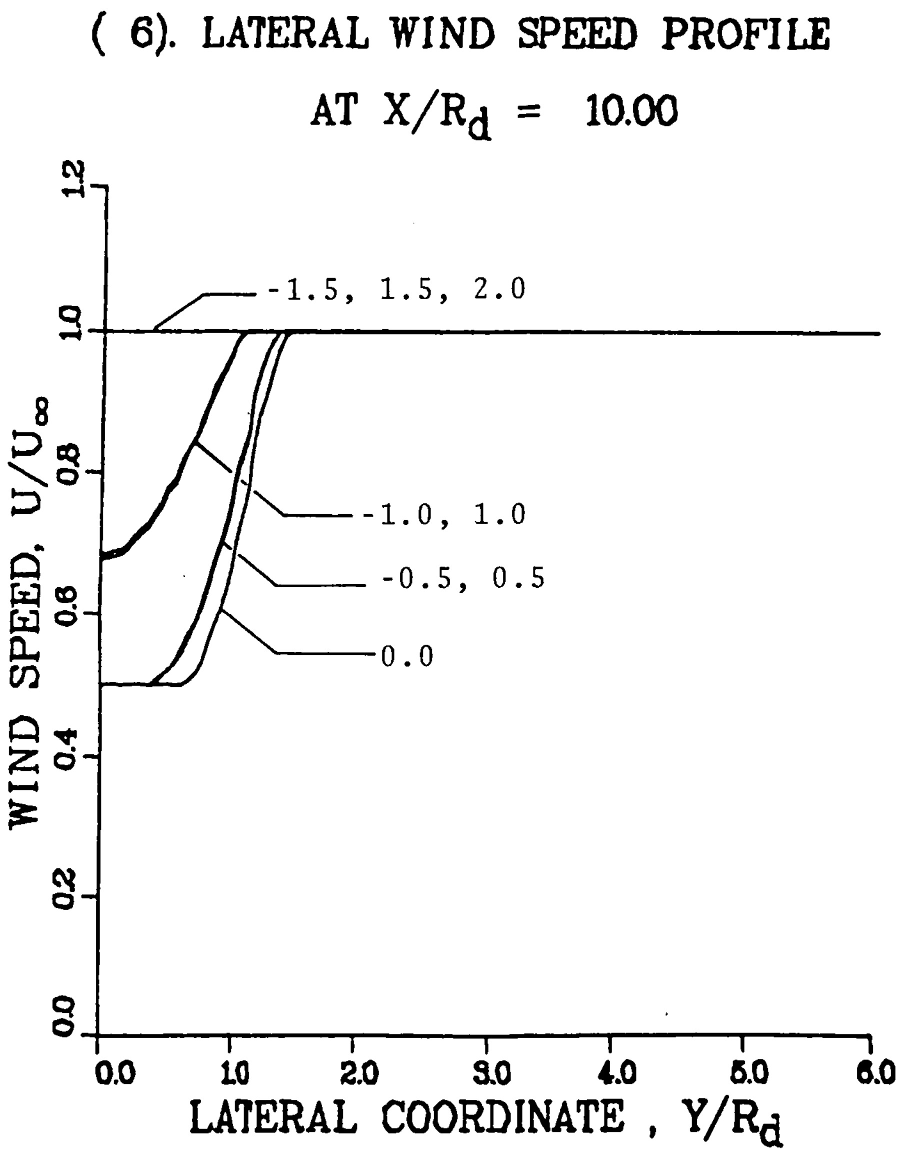


Figure 5-2. Wake plots for Abramovich solution for m = 2.0 (continued).

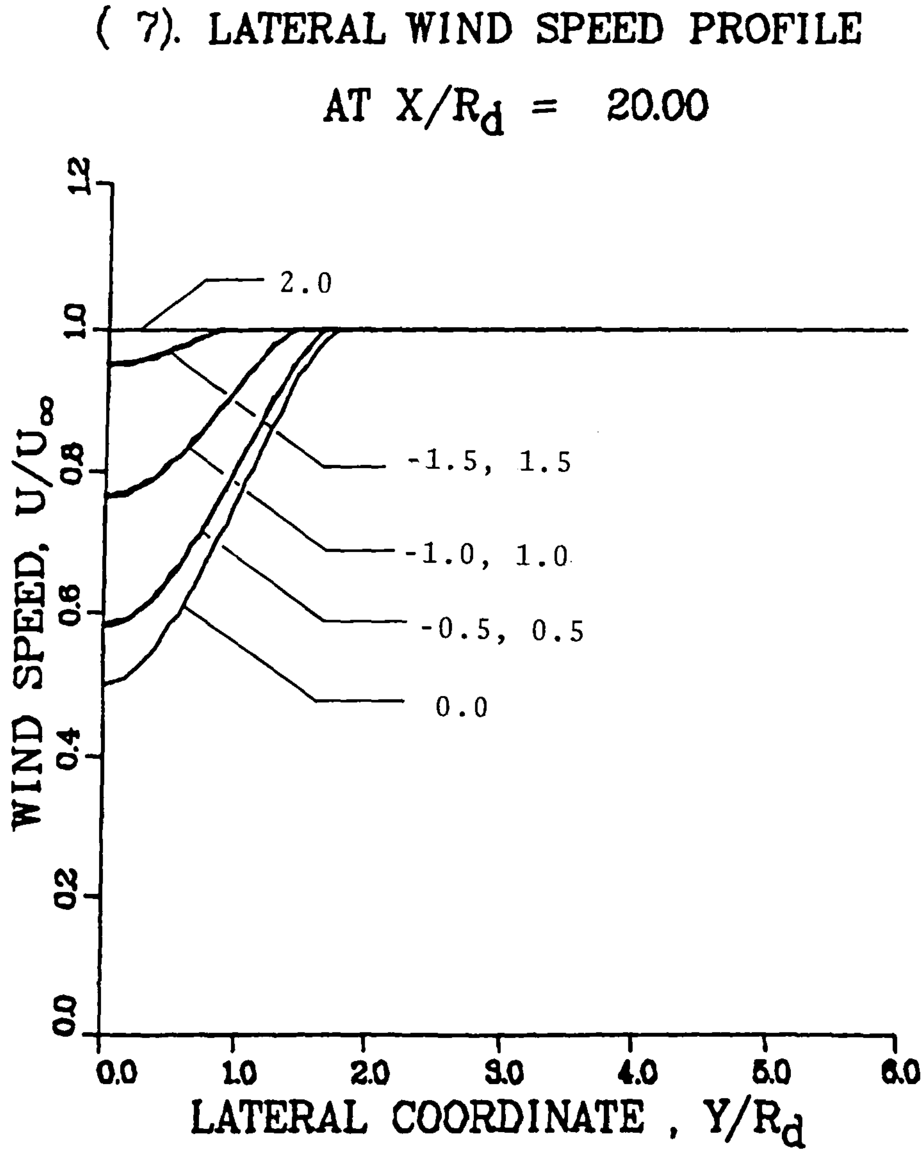


Figure 5-2. Wake plots for Abramovich solution for m = 2.0 (continued).

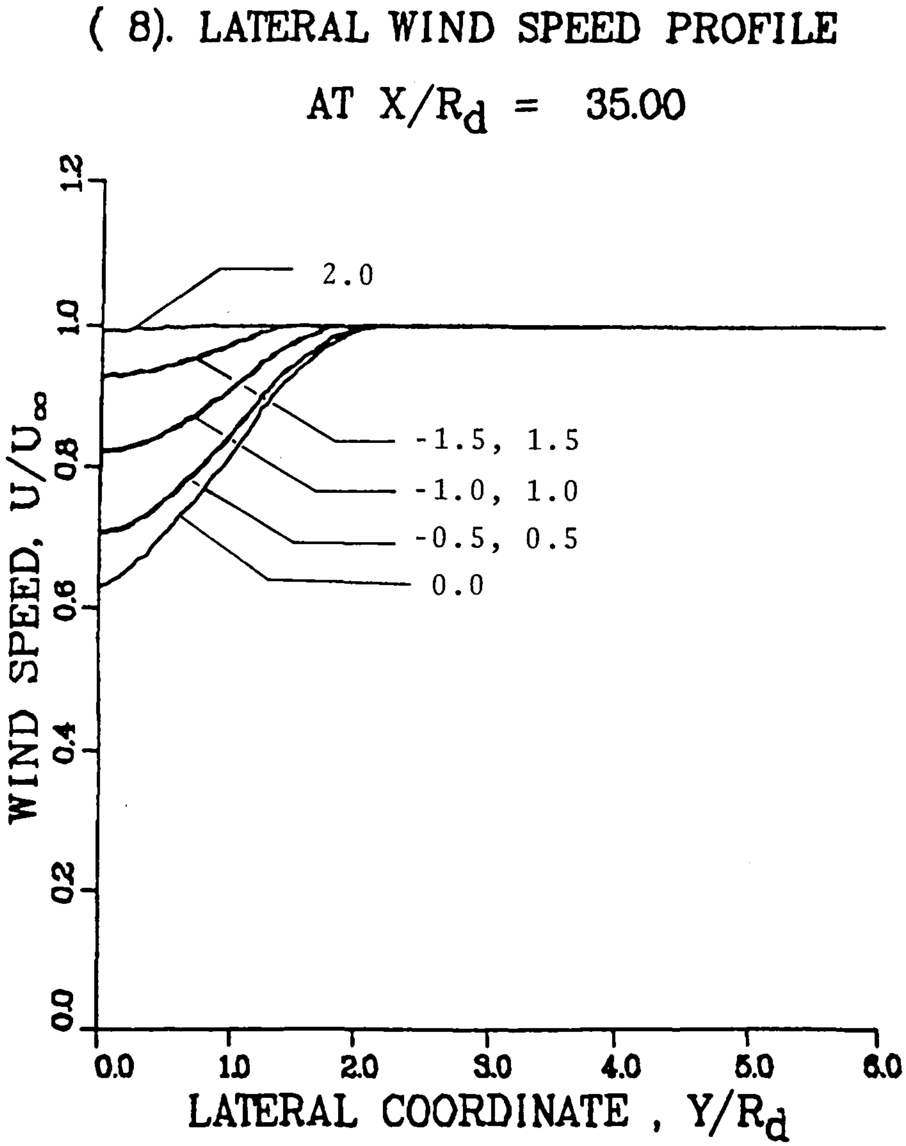


Figure 5-2. Wake plots for Abramovich solution for m = 2.0 (continued).

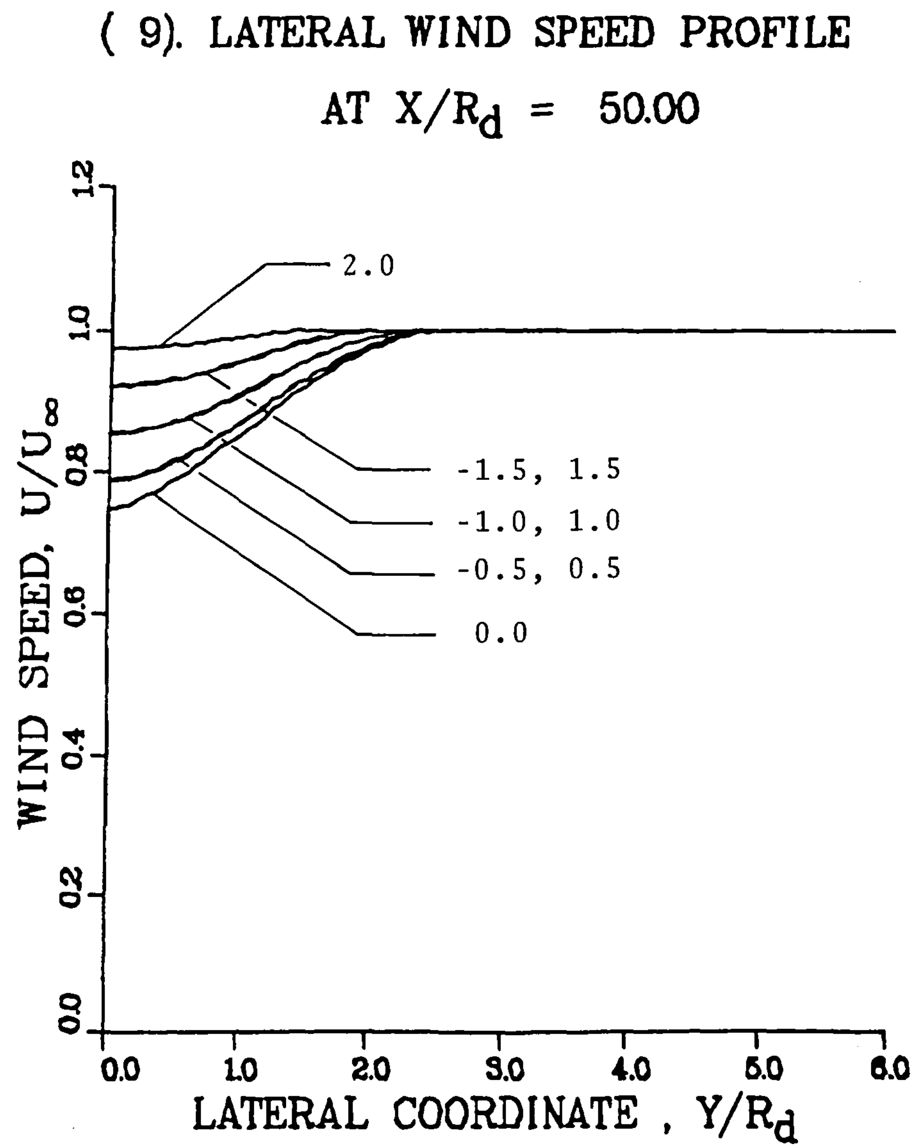


Figure 5-2. Wake plots for Abramovich solution for m = 2.0 (concluded).

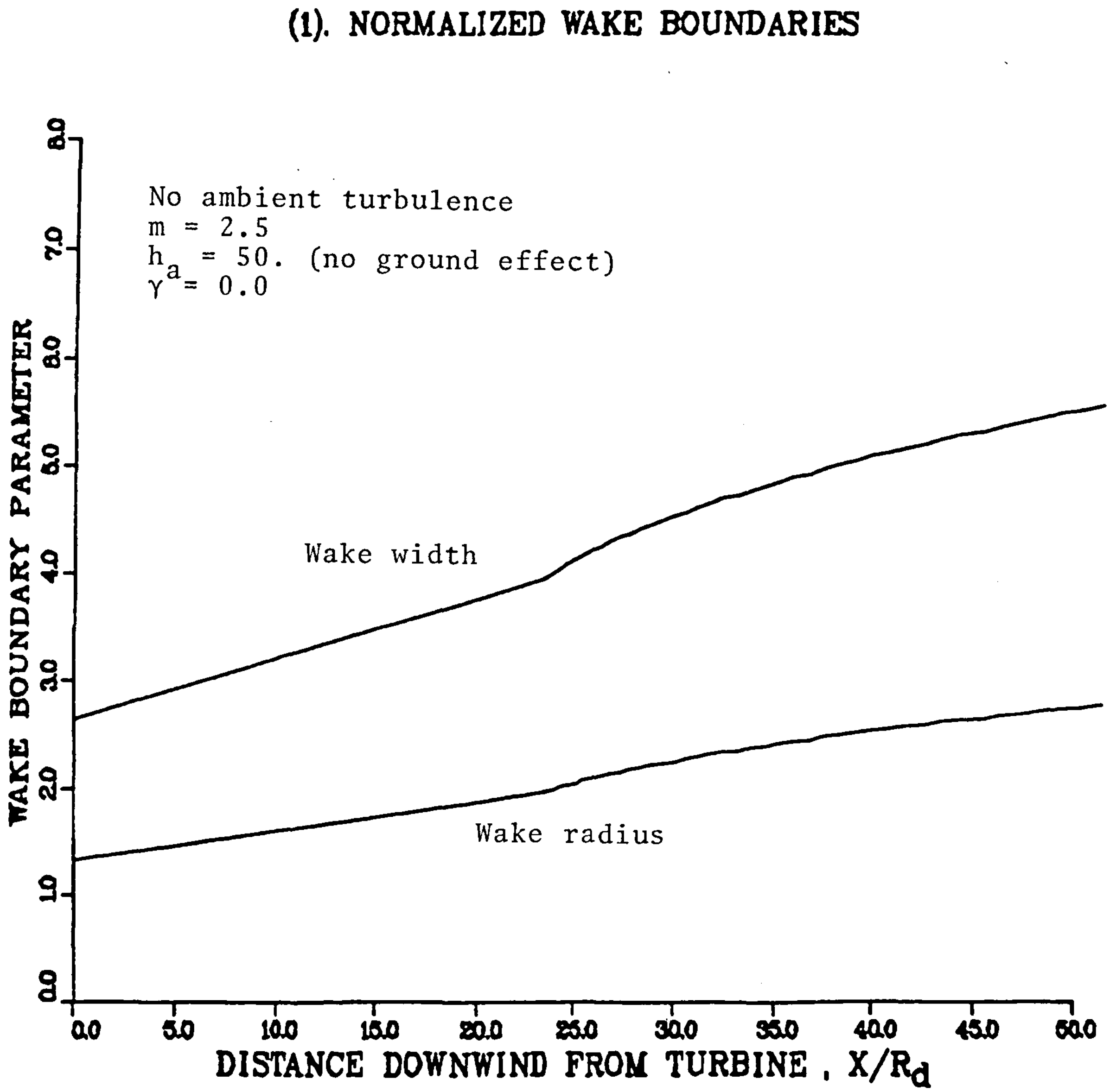


Figure 5-3. Wake plots for Abramovich solution for m = 2.5.

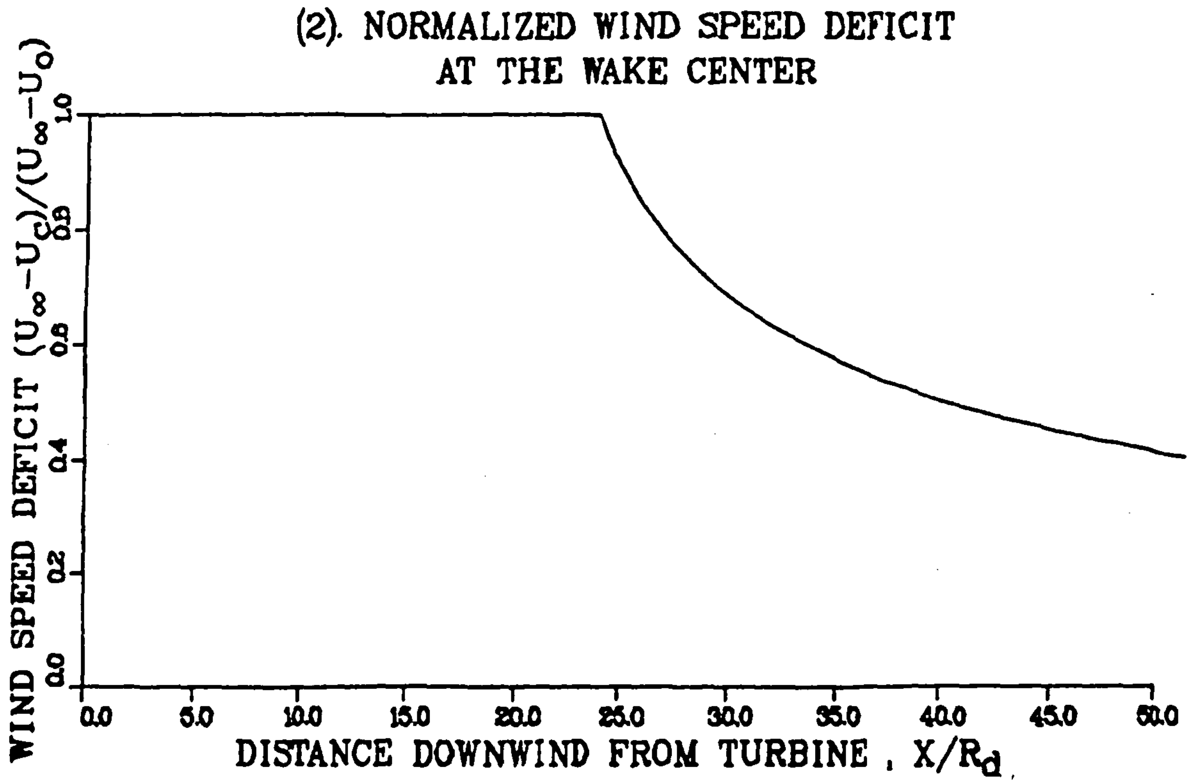


Figure 5-3. Wake plots for Abramovich solution for m = 2.5 (continued).

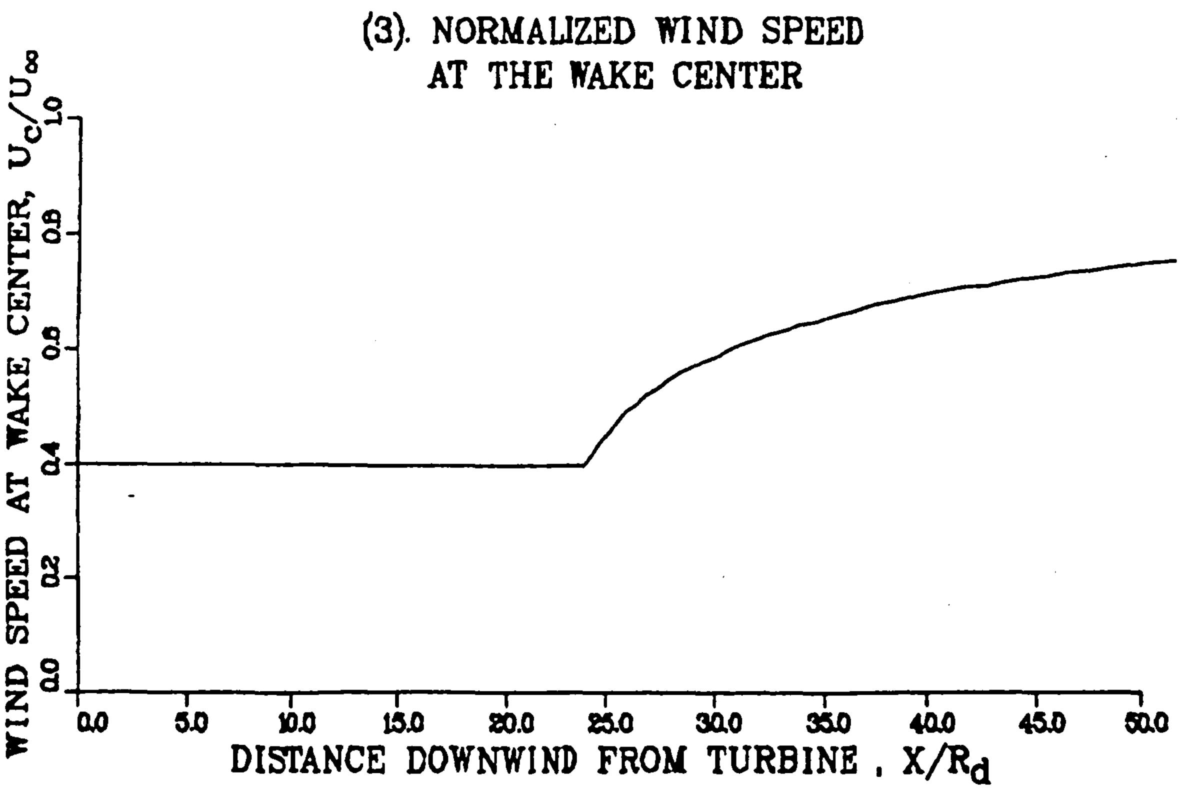


Figure 5-3. Wake plots for Abramovich solution for m = 2.5. (continued).

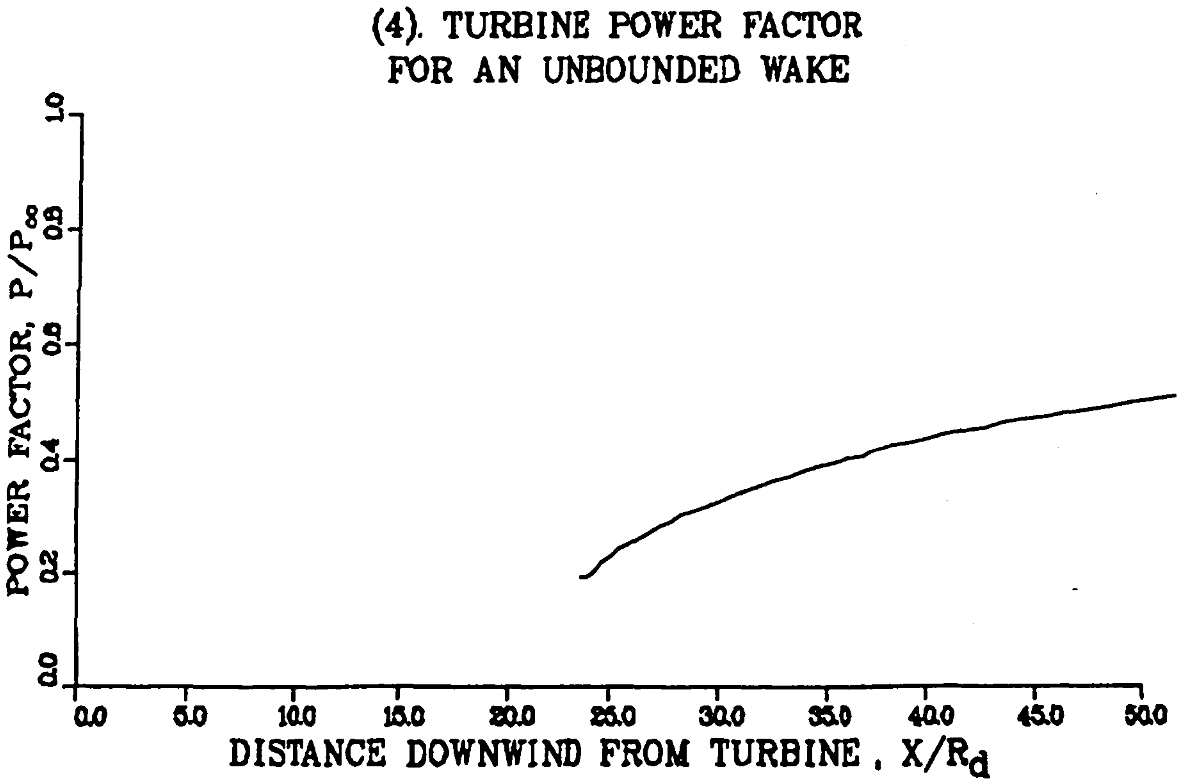


Figure 5-3. Wake plots for Abramovich solution for m = 2.5 (continued).

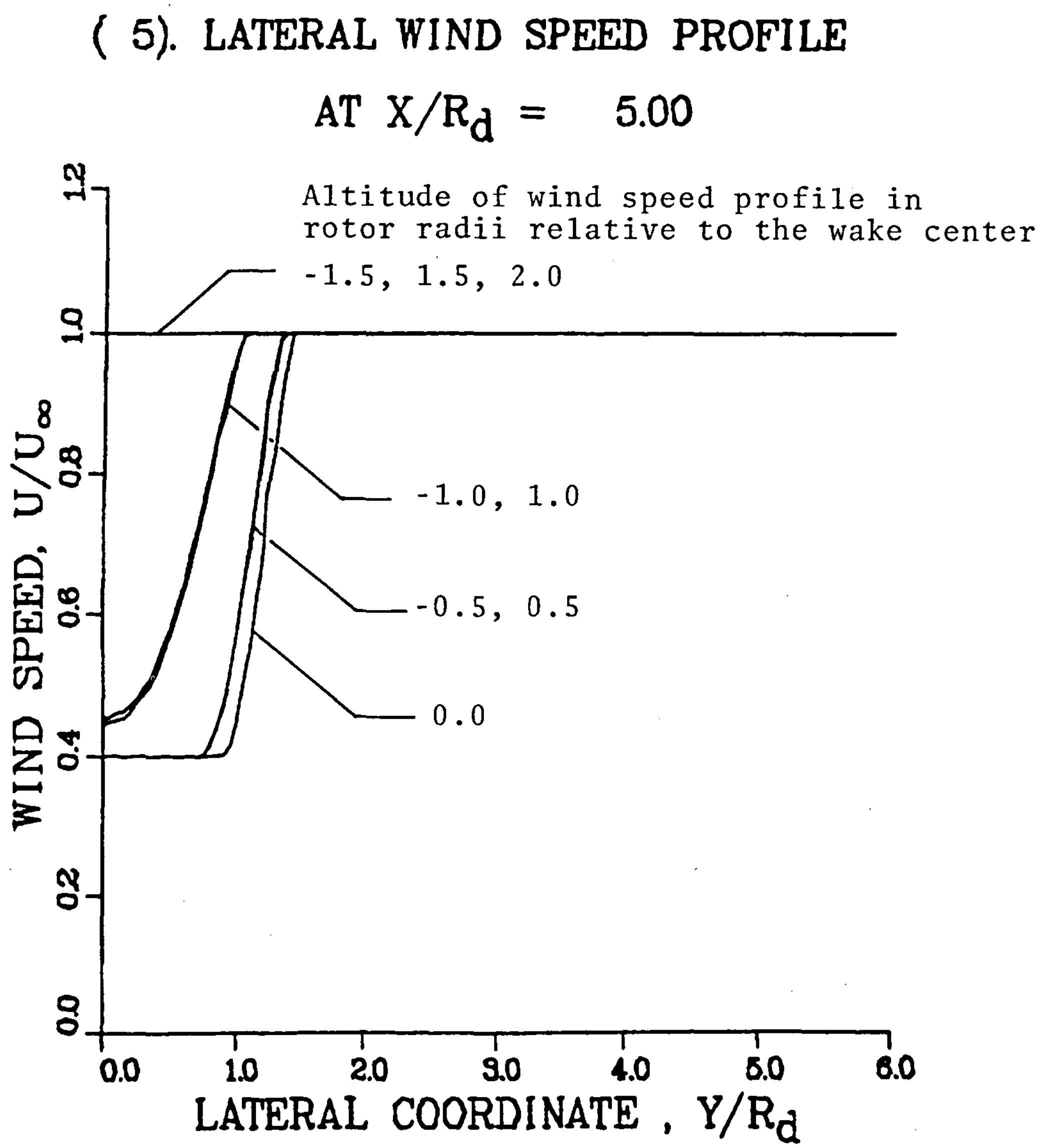


Figure 5-3. Wake plots for Abramovich solution for m = 2.5 (continued).

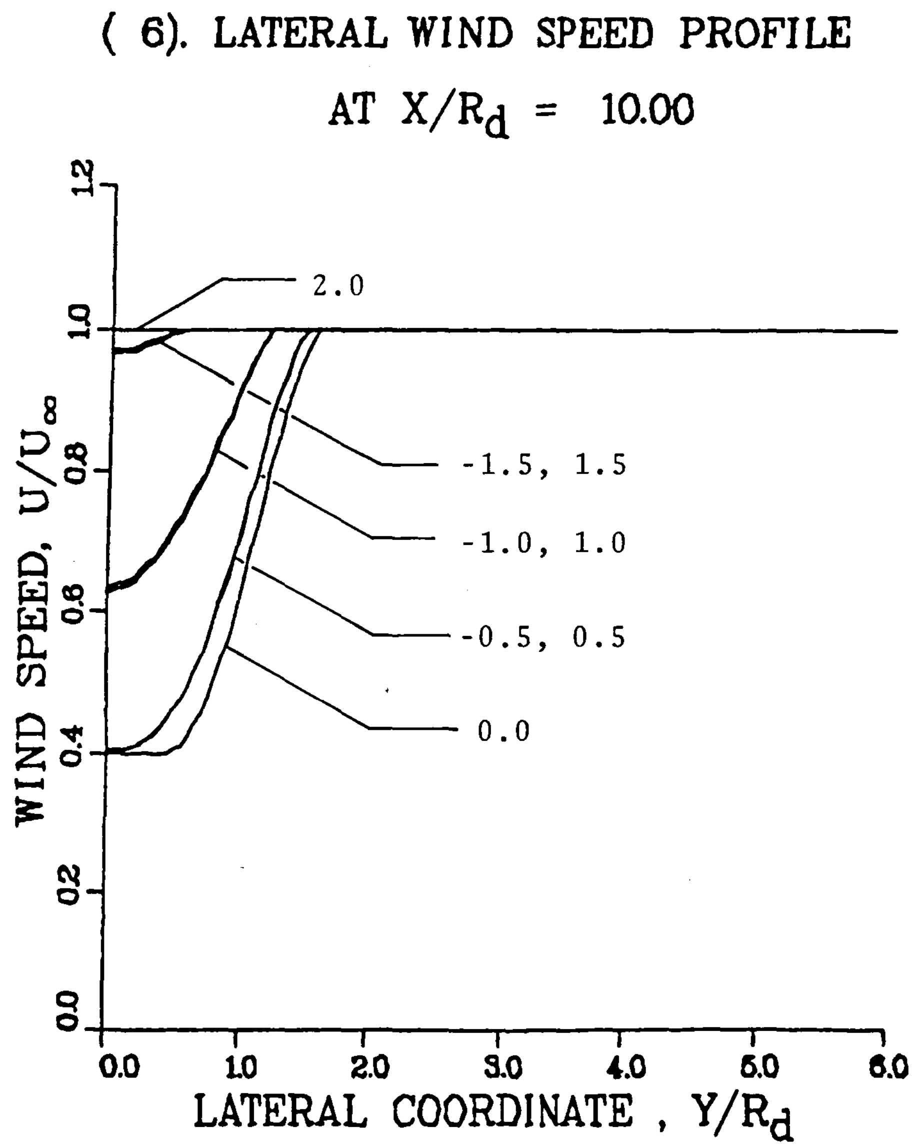


Figure 5-3. Wake plots for Abramovich solution for m = 2.5 (continued).

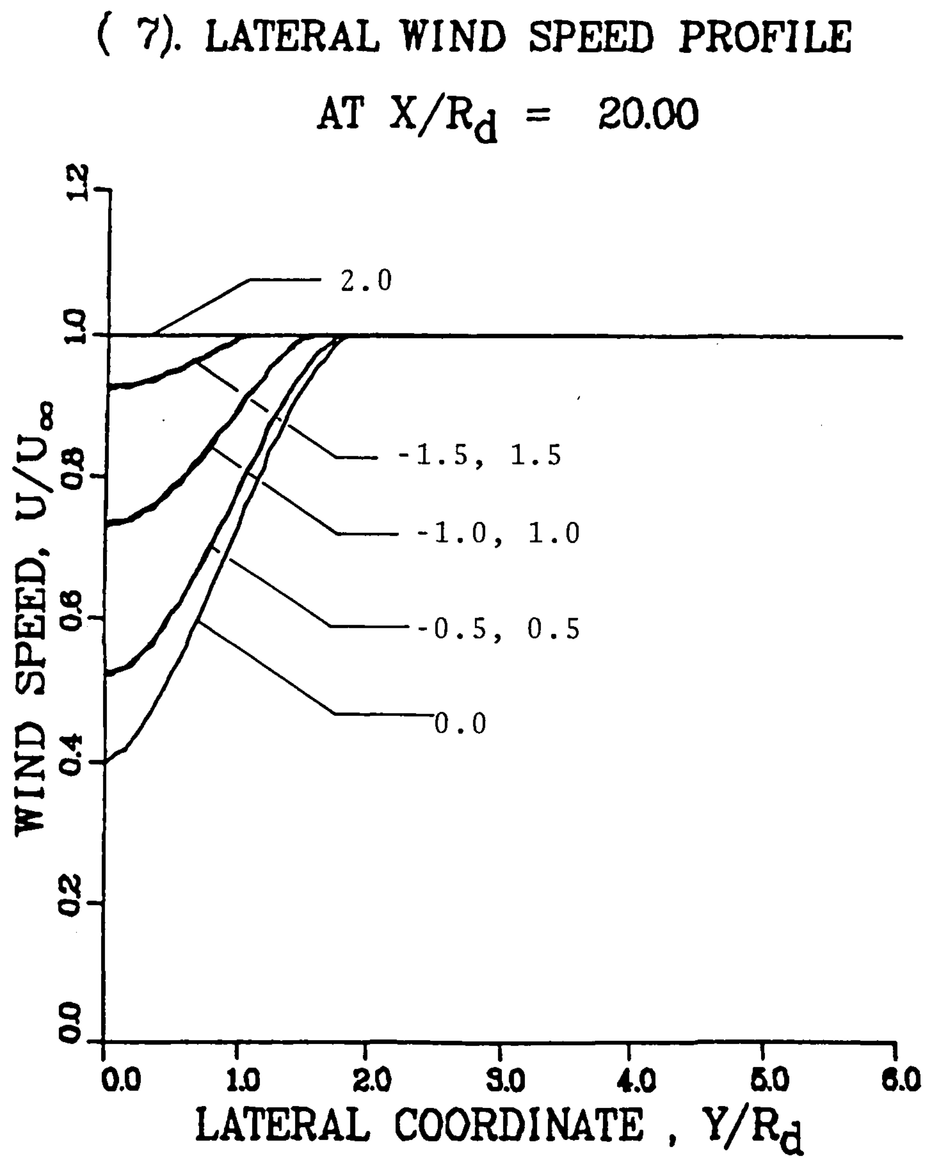


Figure 5-3. Wake plots for Abramovich solution for m = 2.5 (continued).

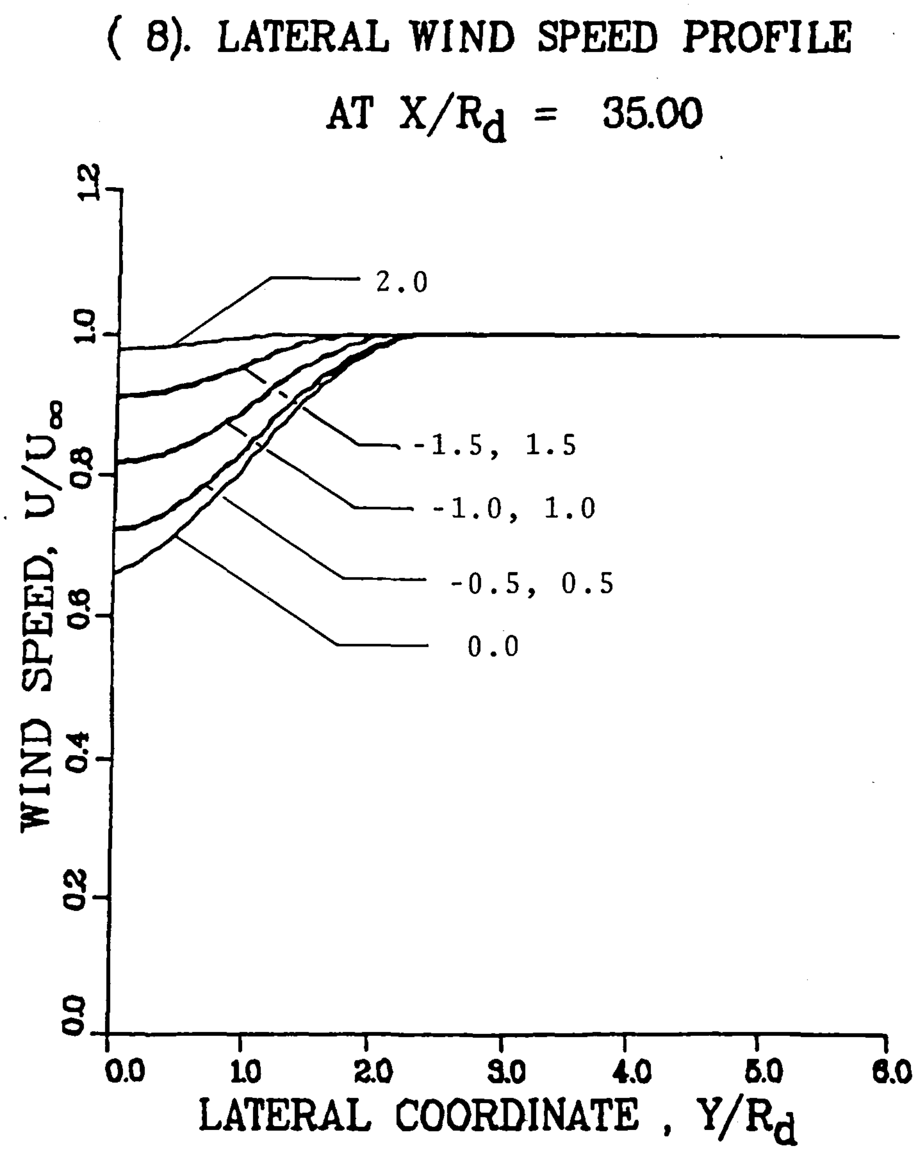


Figure 5-3. Wake plots for Abramovich solution for m = 2.5 (continued).

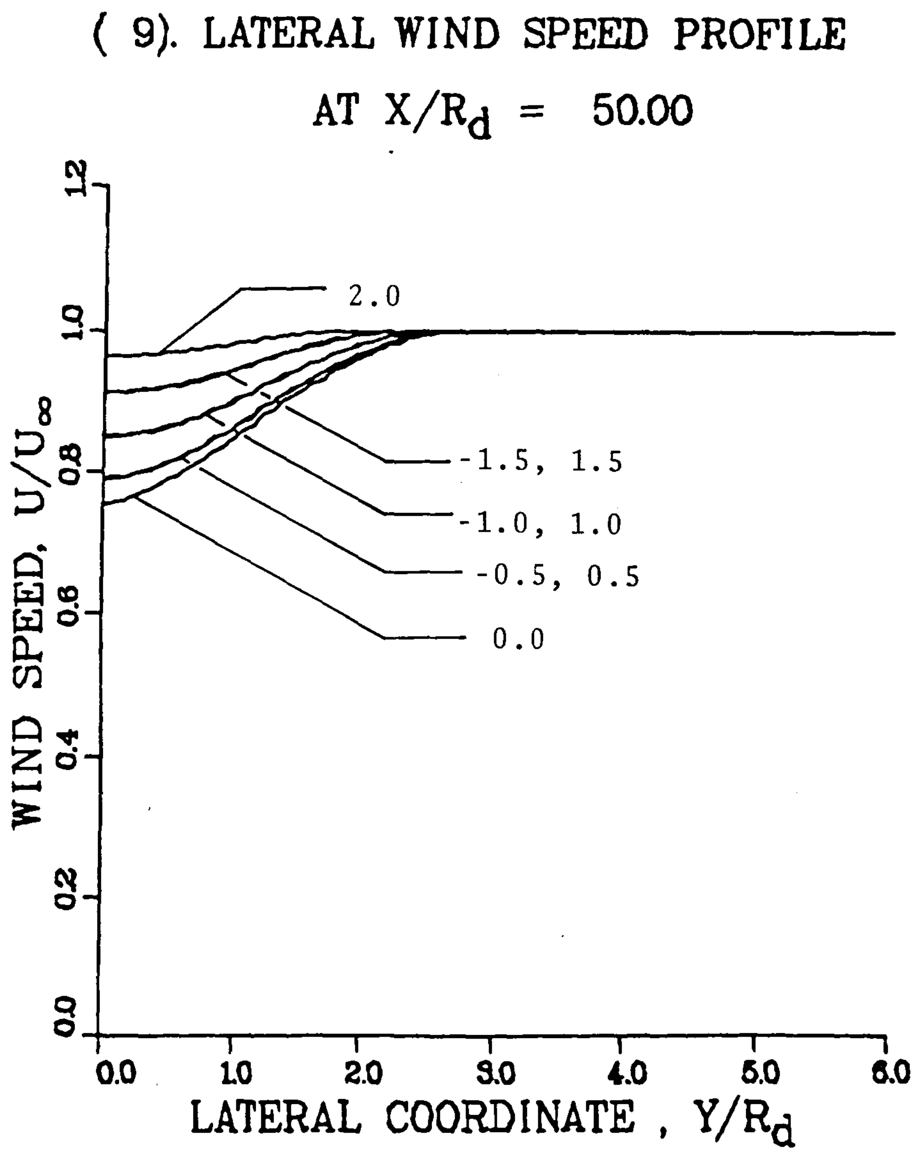


Figure 5-3. Wake plots for Abramovich solution for m = 2.5 (concluded).

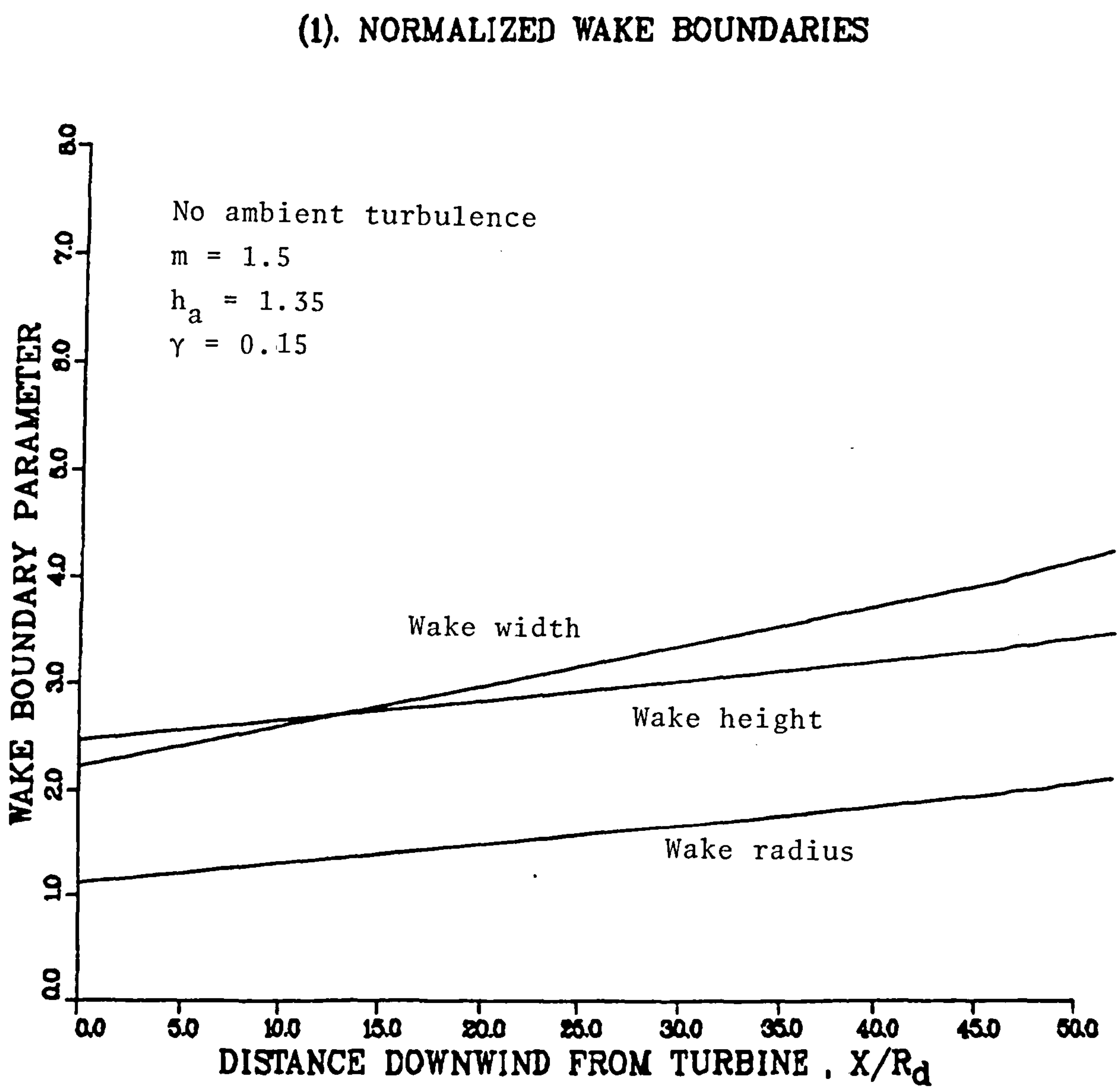


Figure 5-4. Wake plots for wake in ground effect with no ambient turbulence.

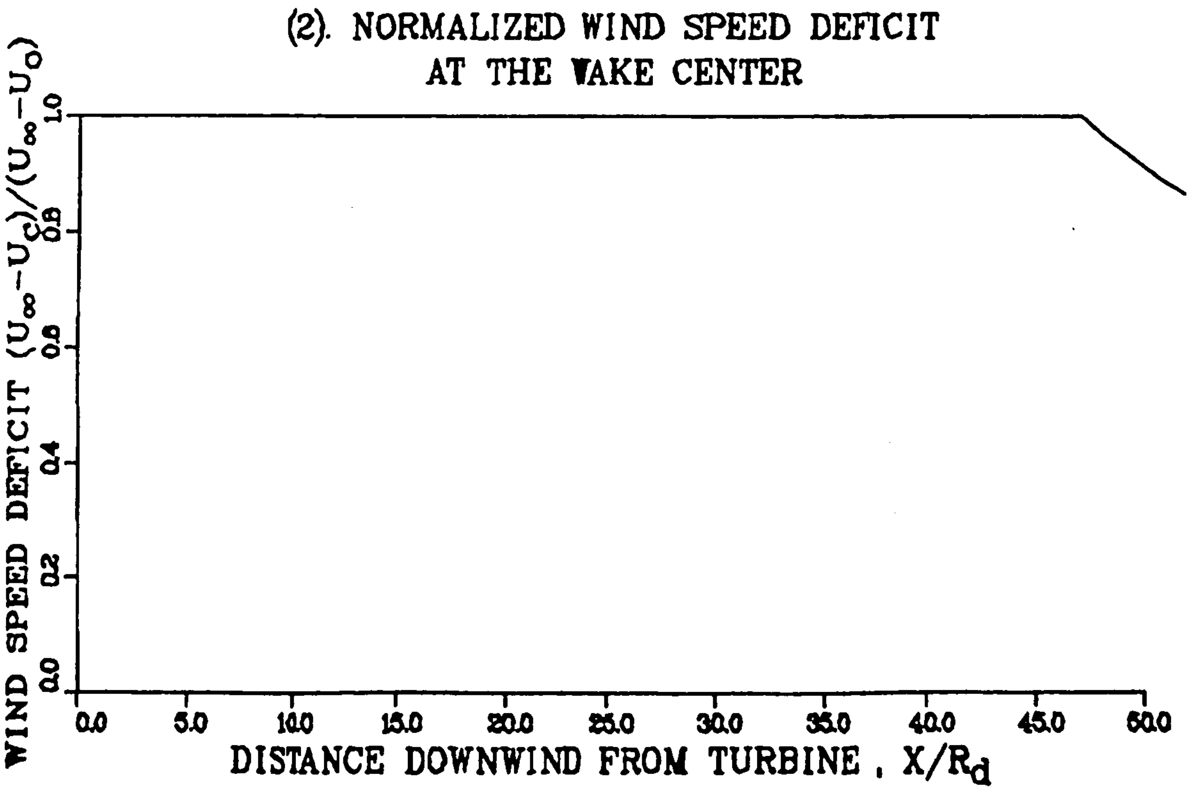


Figure 5-4. Wake plots for wake in ground effect with no ambient turbulence (continued).

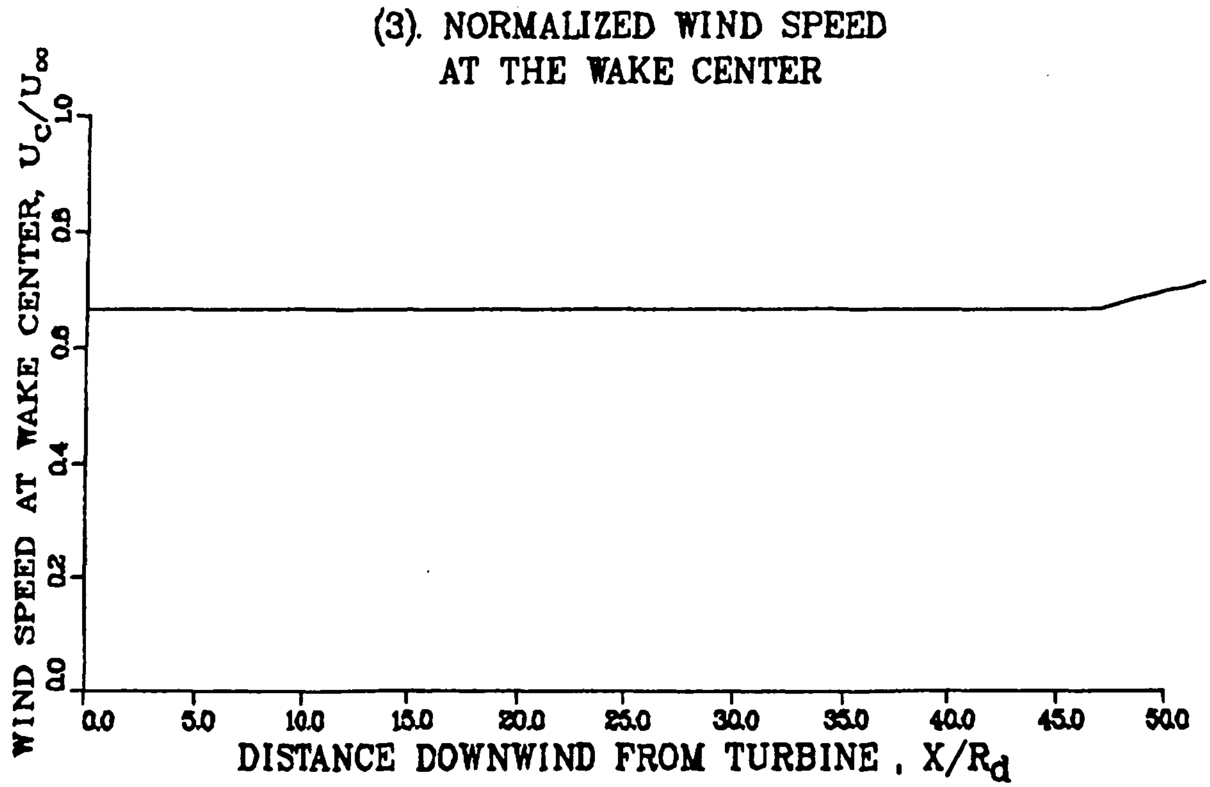


Figure 5-4. Wake plots for wake in ground effect with no ambient turbulence (continued).

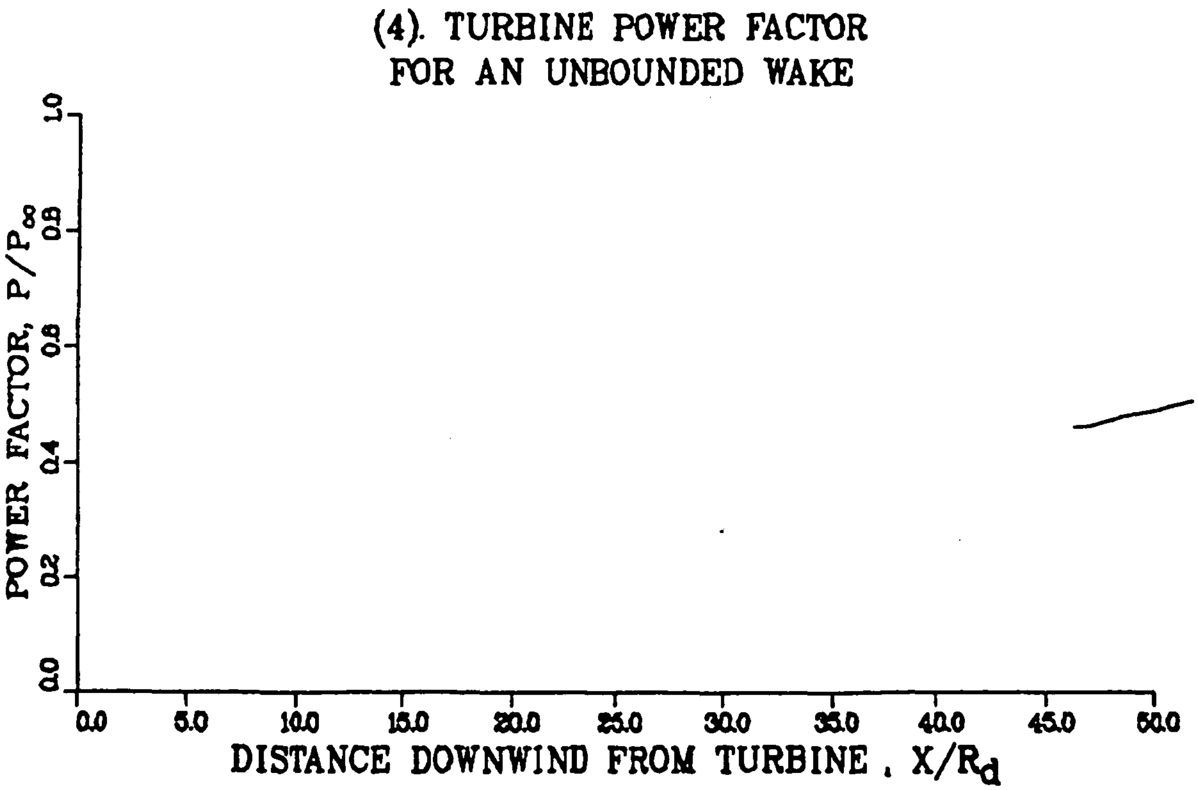


Figure 5-4. Wake plots for wake in ground effect with no ambient turbulence (continued).

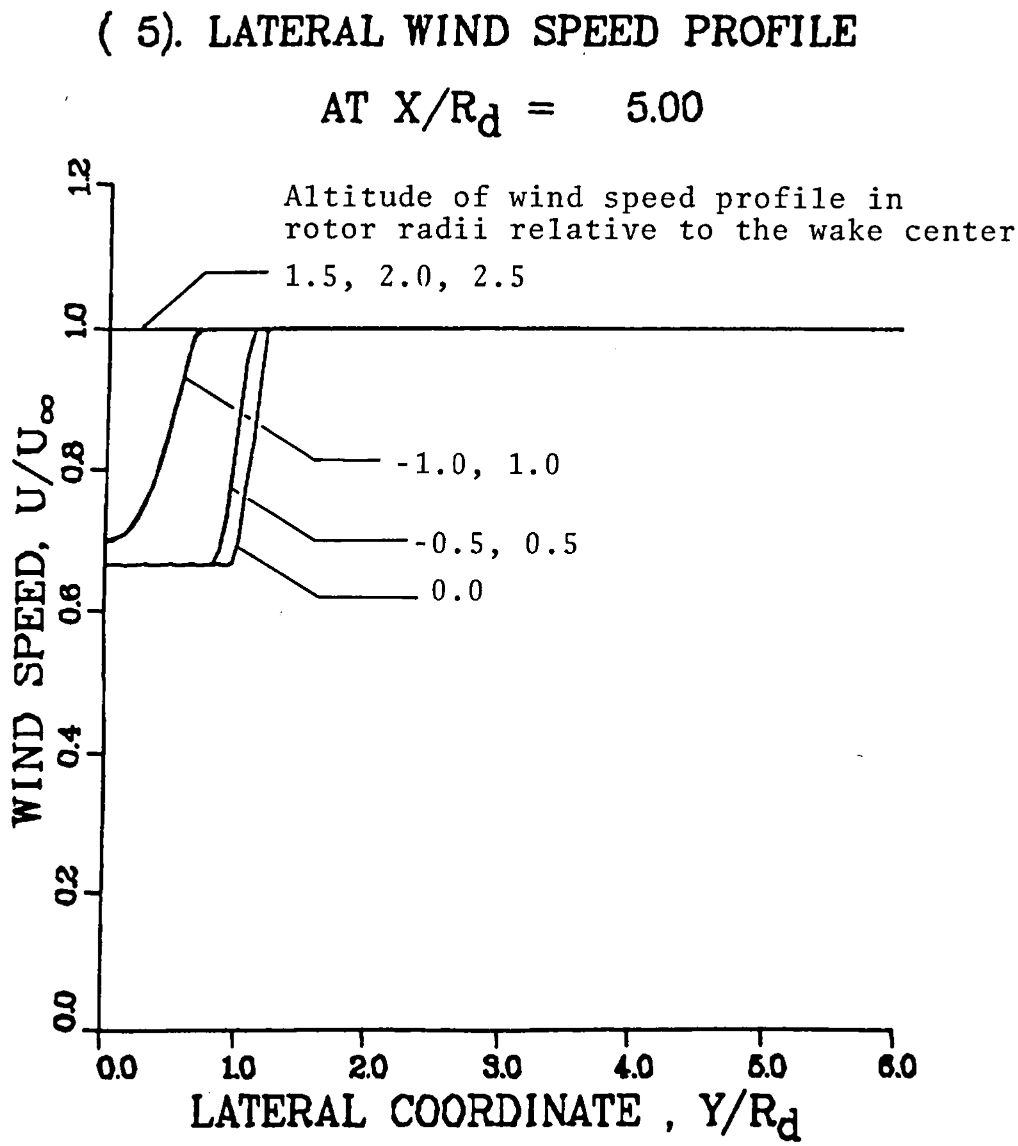


Figure 5-4. Wake plots for wake in ground effect with no ambient turbulence (continued).

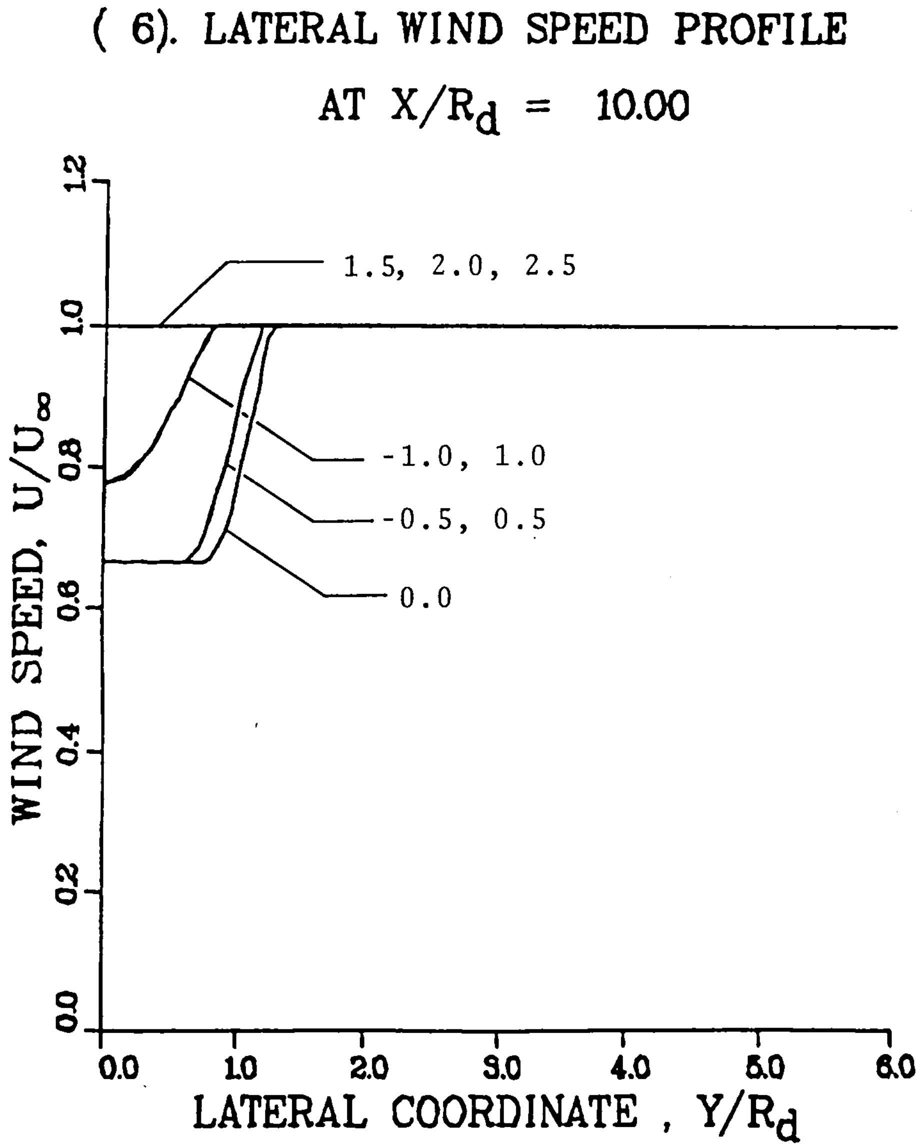


Figure 5-4. Wake plots for wake in ground effect with no ambient turbulence (continued).

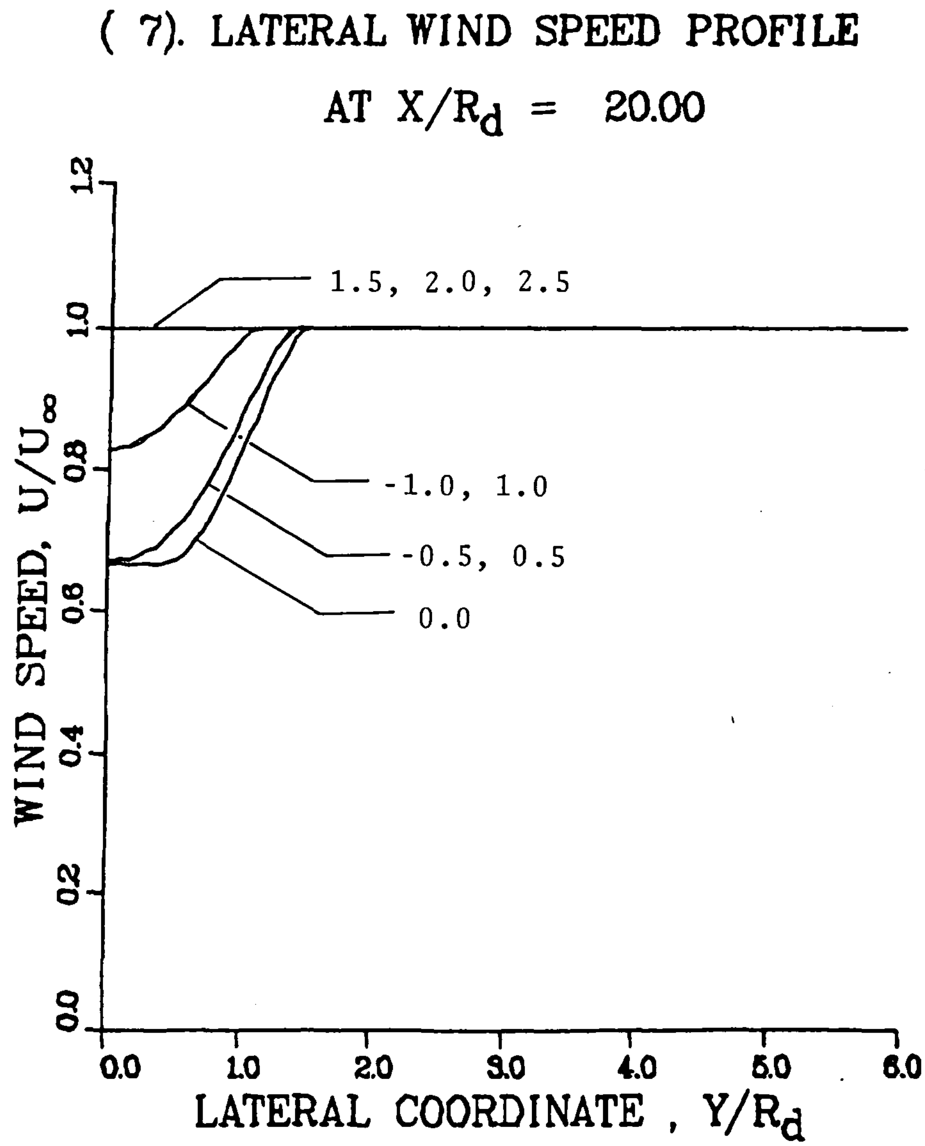


Figure 5-4. Wake plots for wake in ground effect with no ambient turbulence (continued).

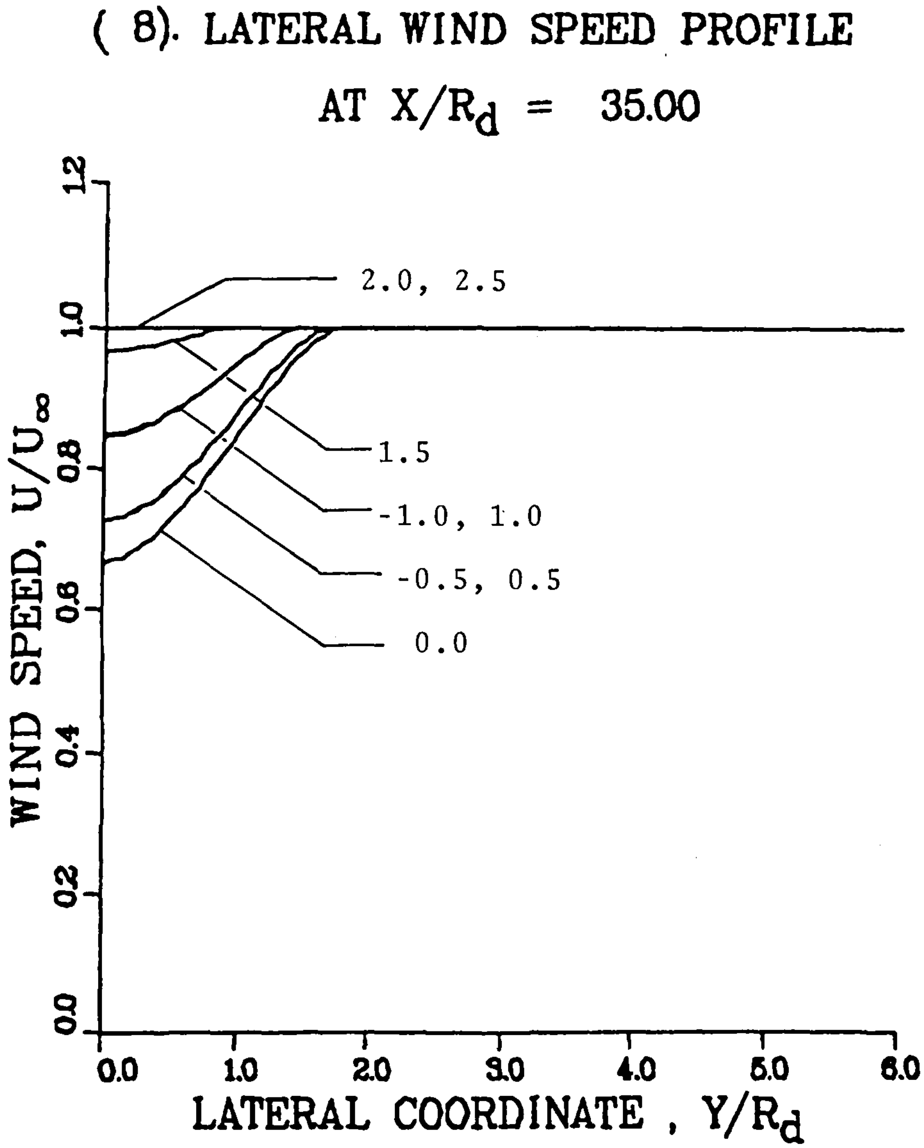


Figure 5-4. Wake plots for wake in ground effect with no ambient turbulence (continued).

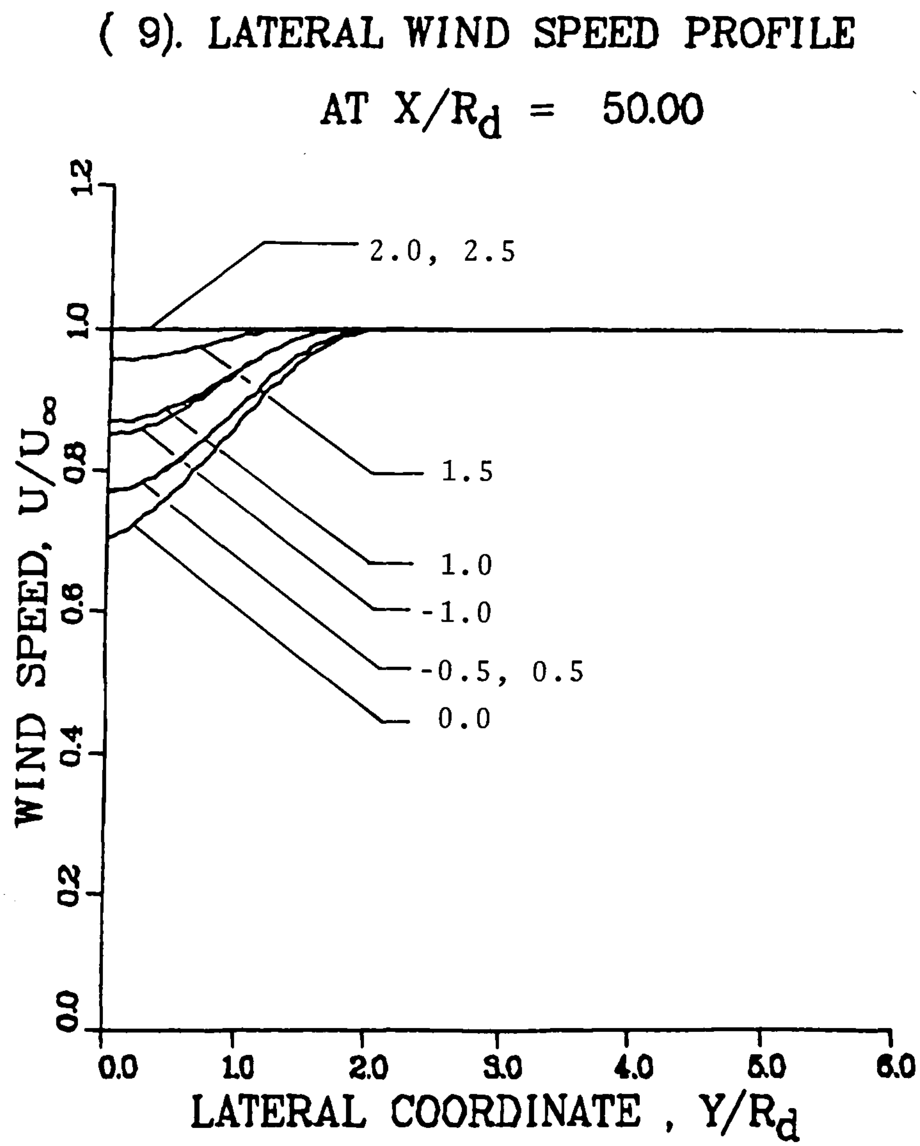


Figure 5-4. Wake plots for wake in ground effect with no ambient turbulence (continued).

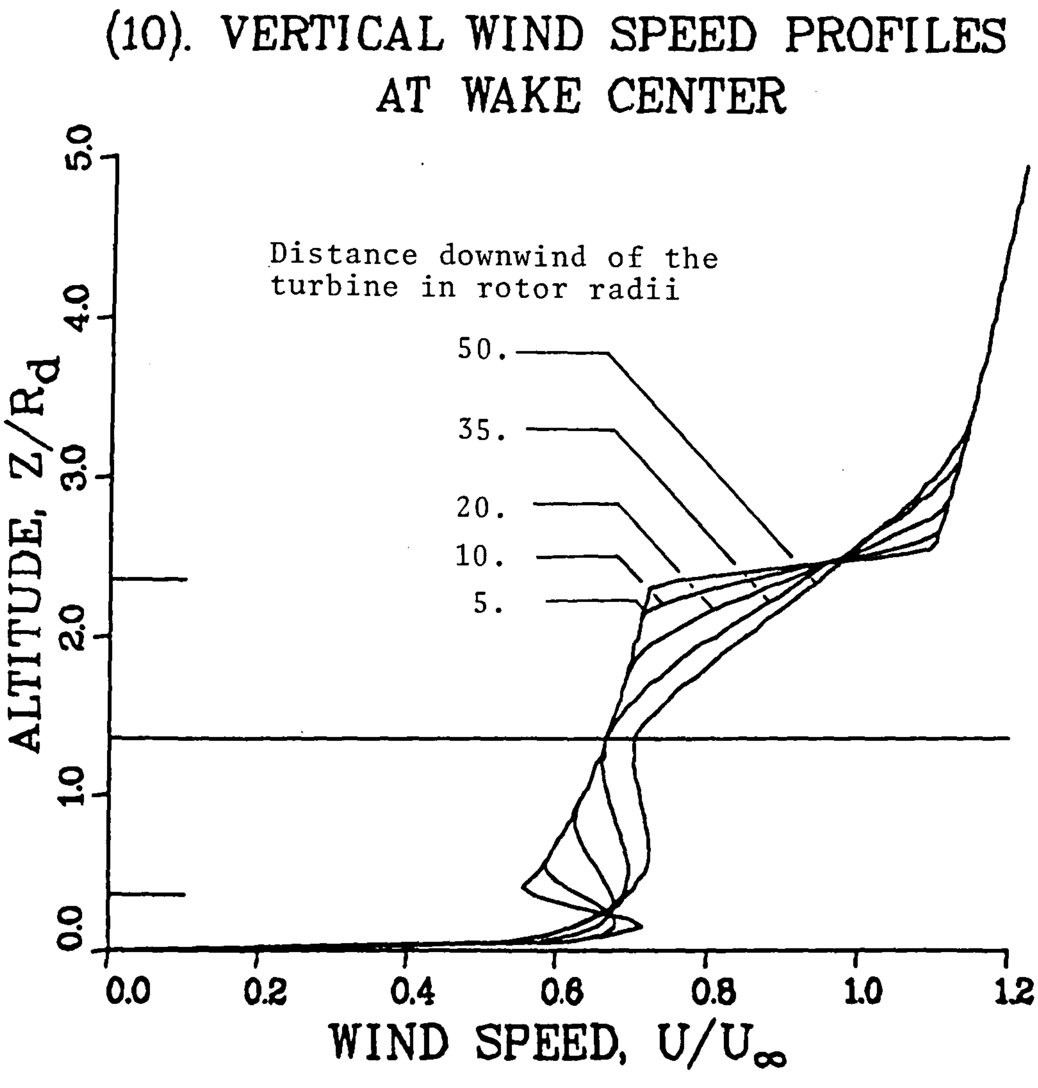


Figure 5-4. Wake plots for wake in ground effect with no ambient turbulence (concluded).

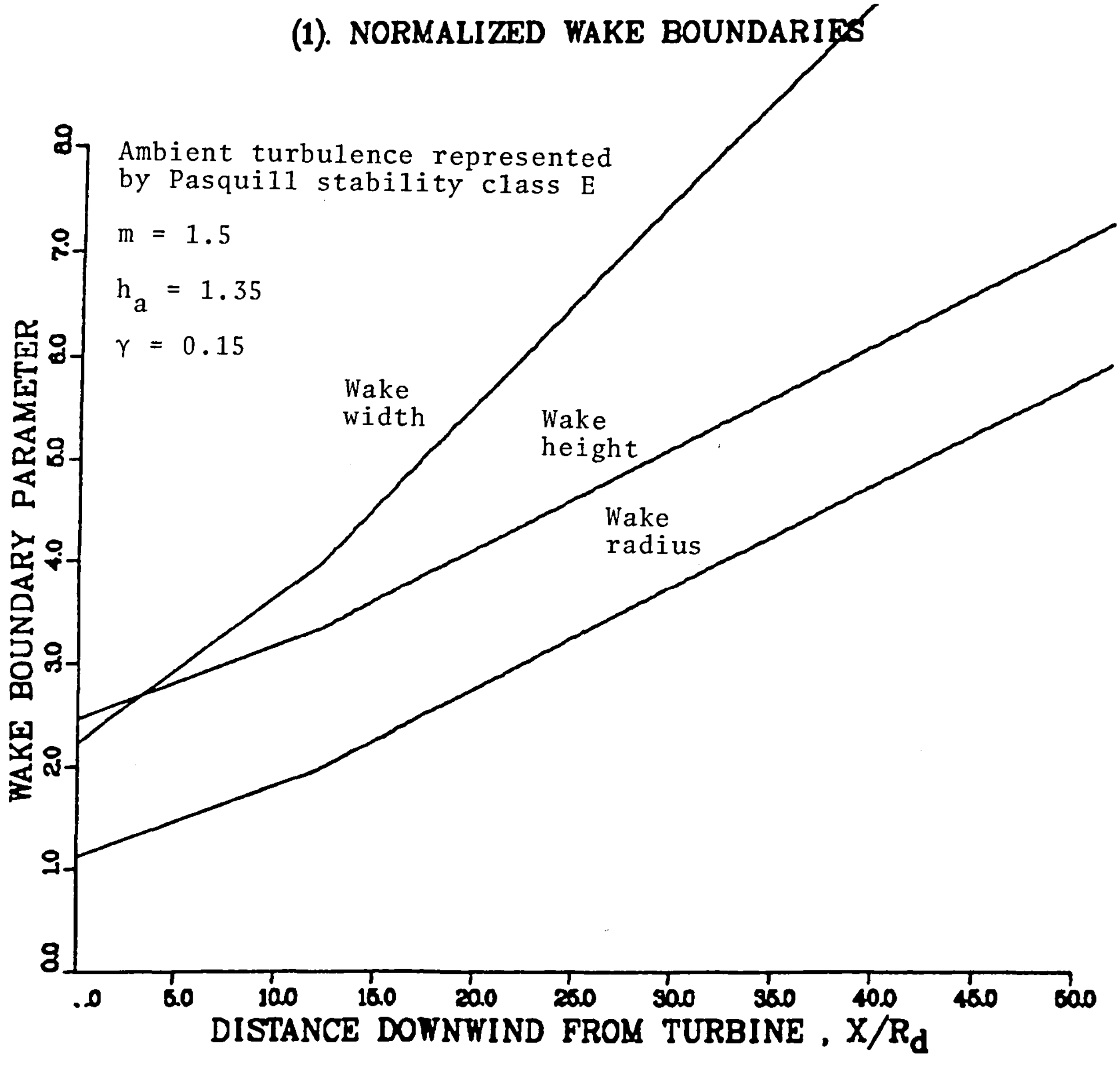


Figure 5-5. Wake plots for wake in ground effect for Pasquill stability class E.

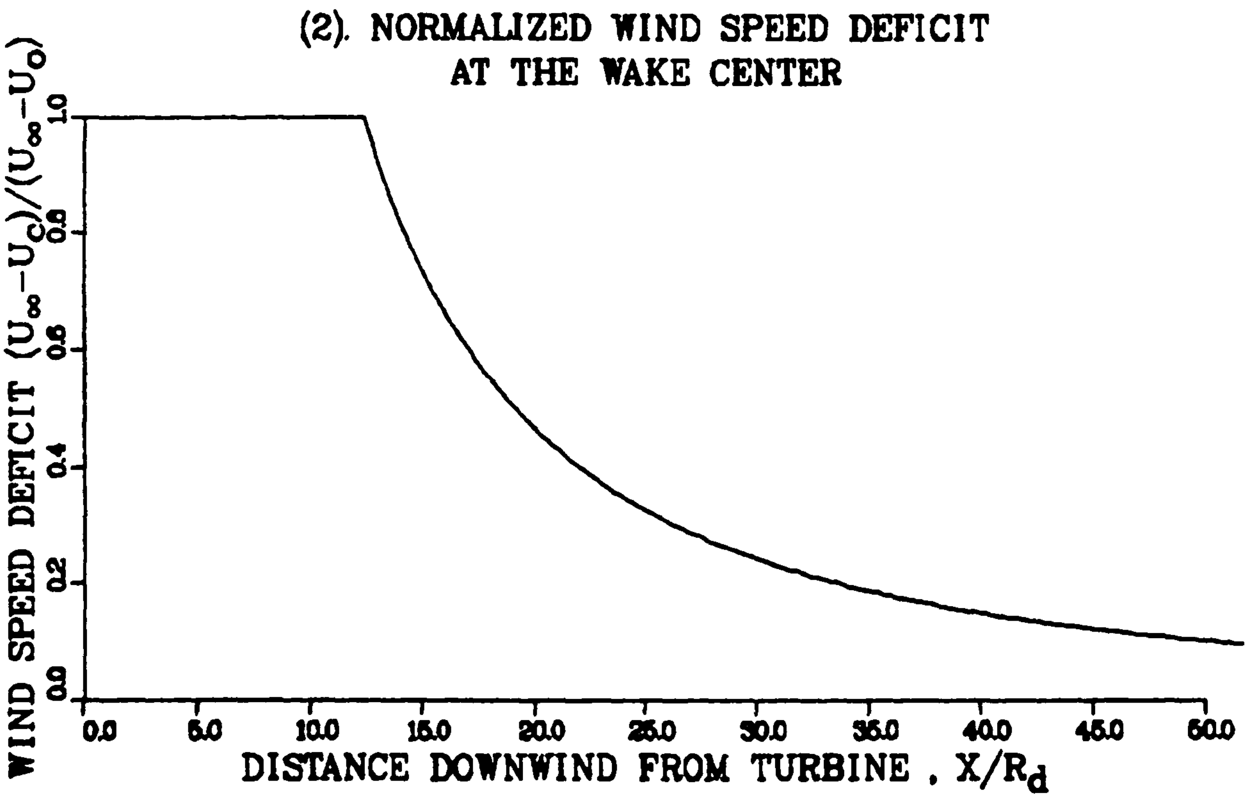


Figure 5-5. Wake plots for wake in ground effect for Pasquill stability class E (continued).

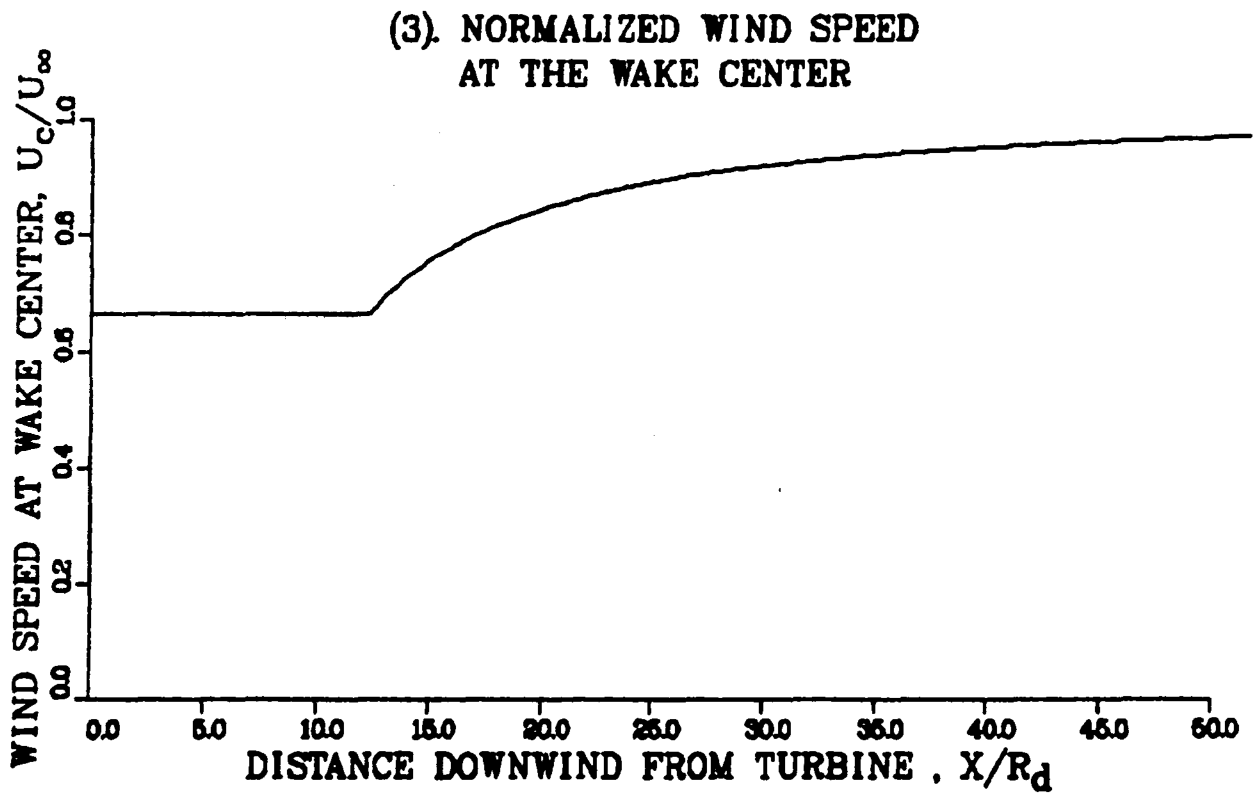


Figure 5-5. Wake plots for wake in ground effect for Pasquill stability class E (continued).

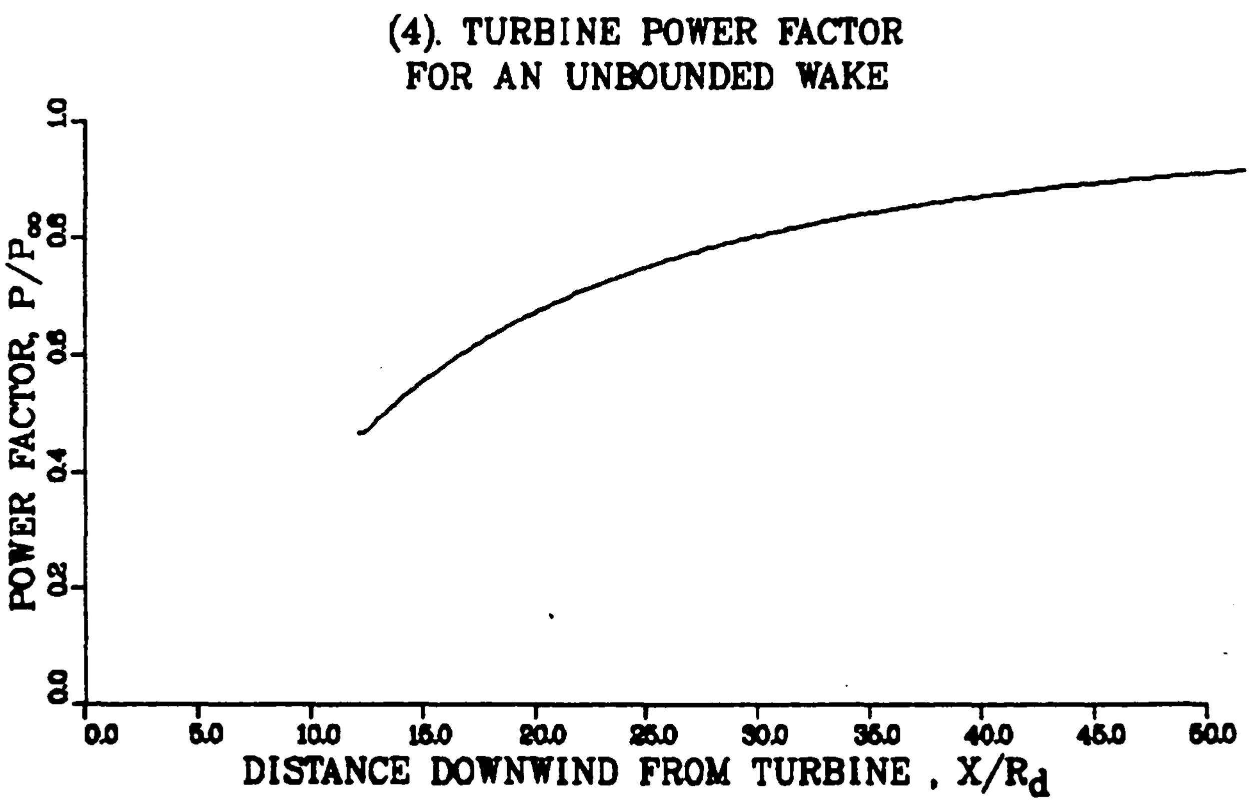


Figure 5-5. Wake plots for wake in ground effect for Pasquill stability class E (continued).

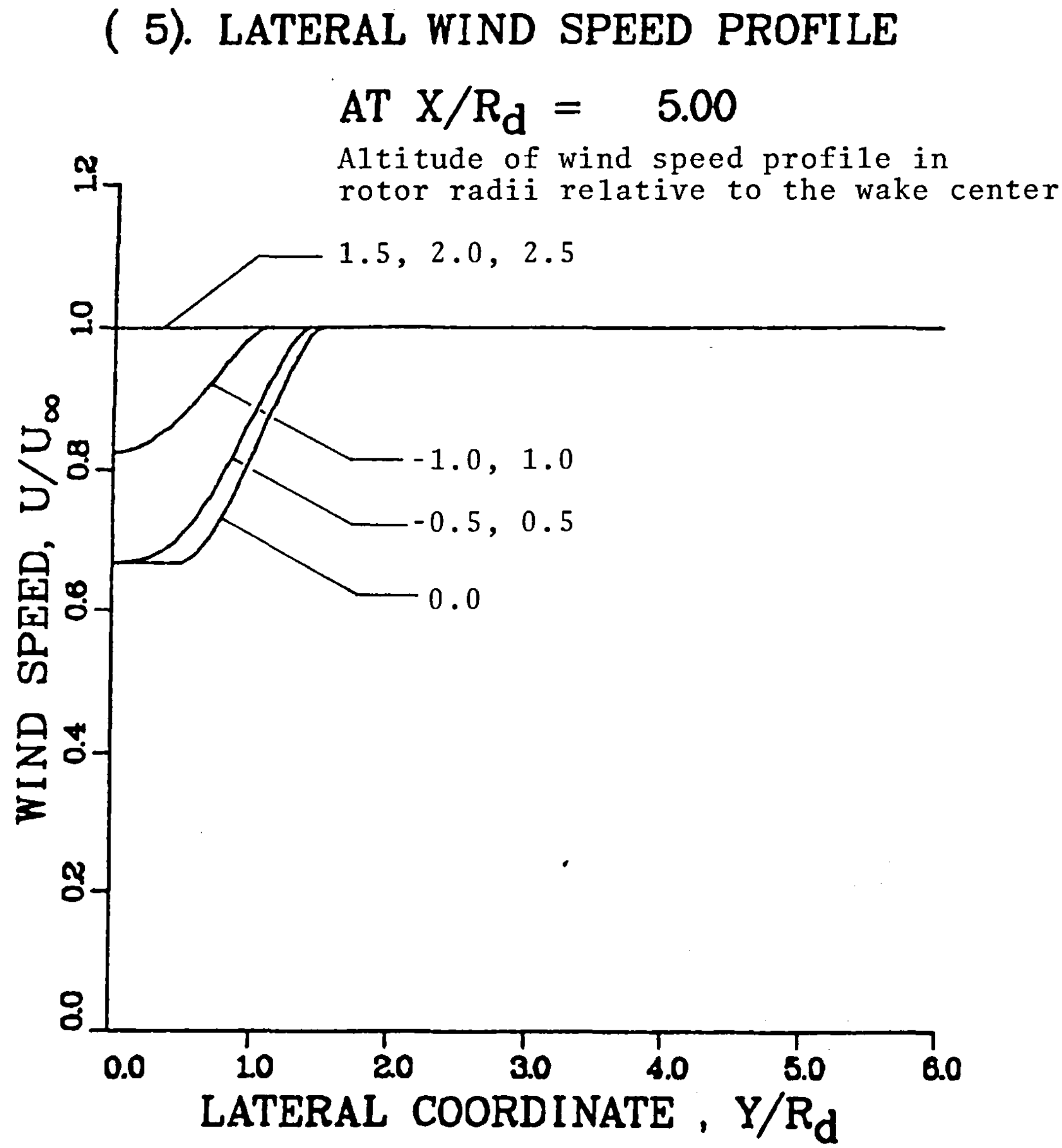


Figure 5-5. Wake plots for wake in ground effect for Pasquill stability class E (continued).

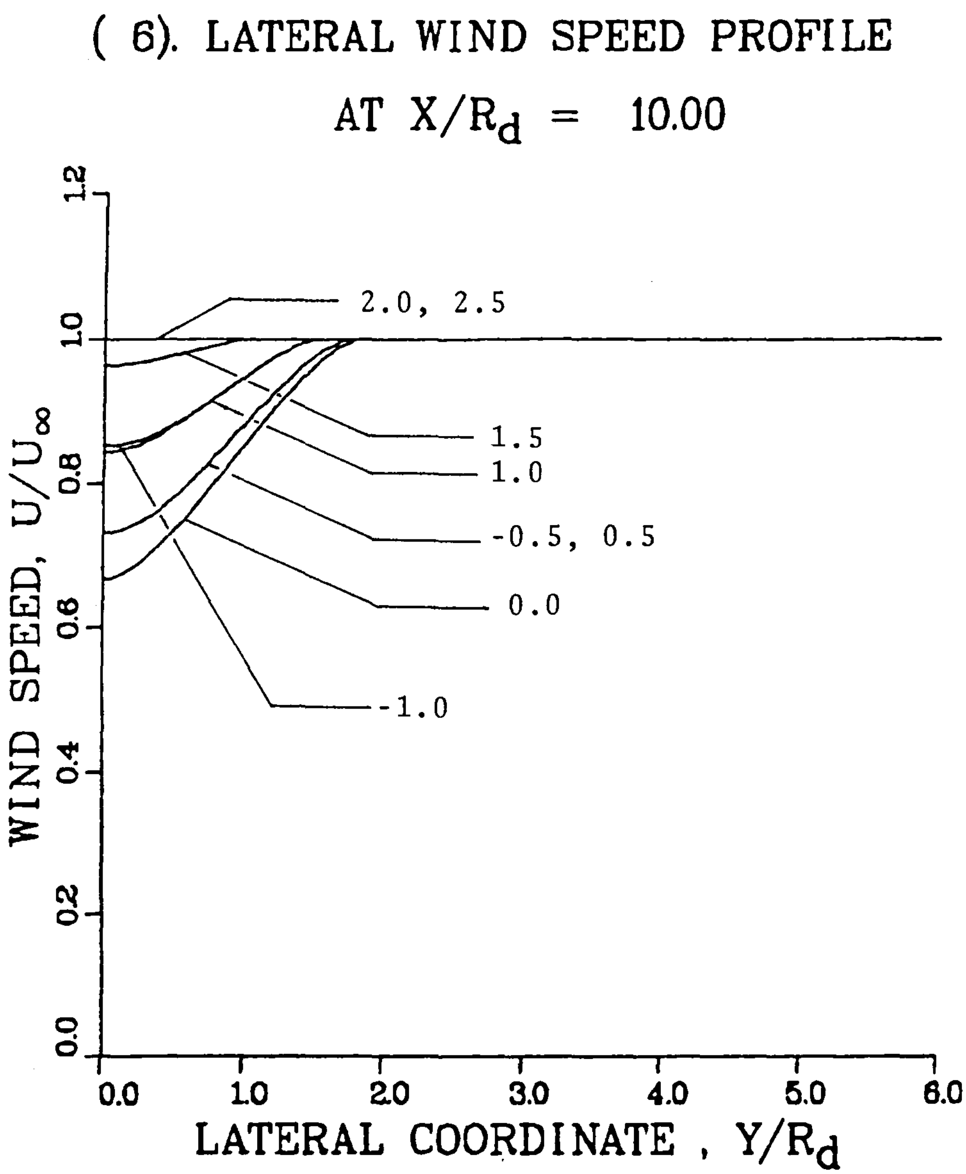


Figure 5-5. Wake plots for wake in ground effect for Pasquill stability class E (continued).

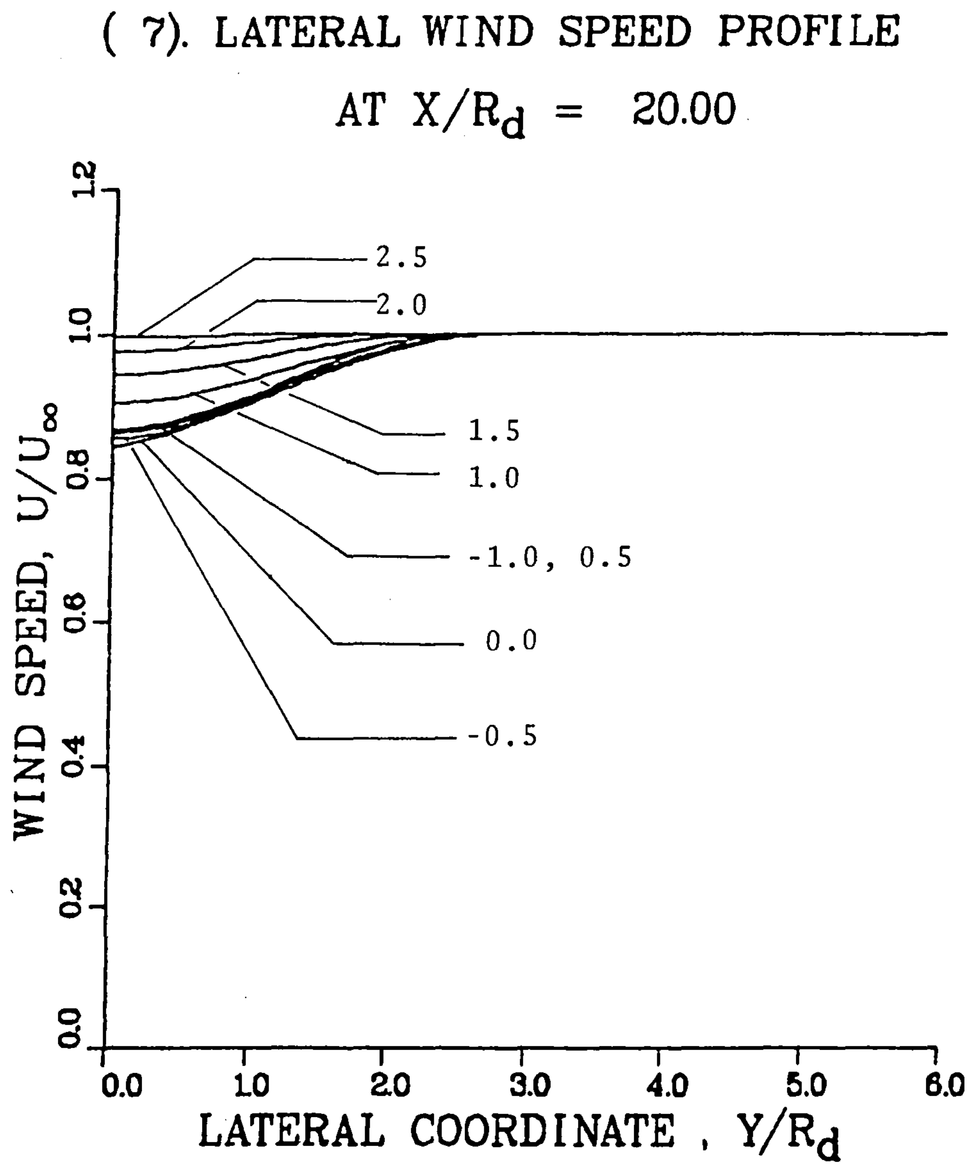


Figure 5-5. Wake plots for wake in ground effect for Pasquill stability class E (continued).

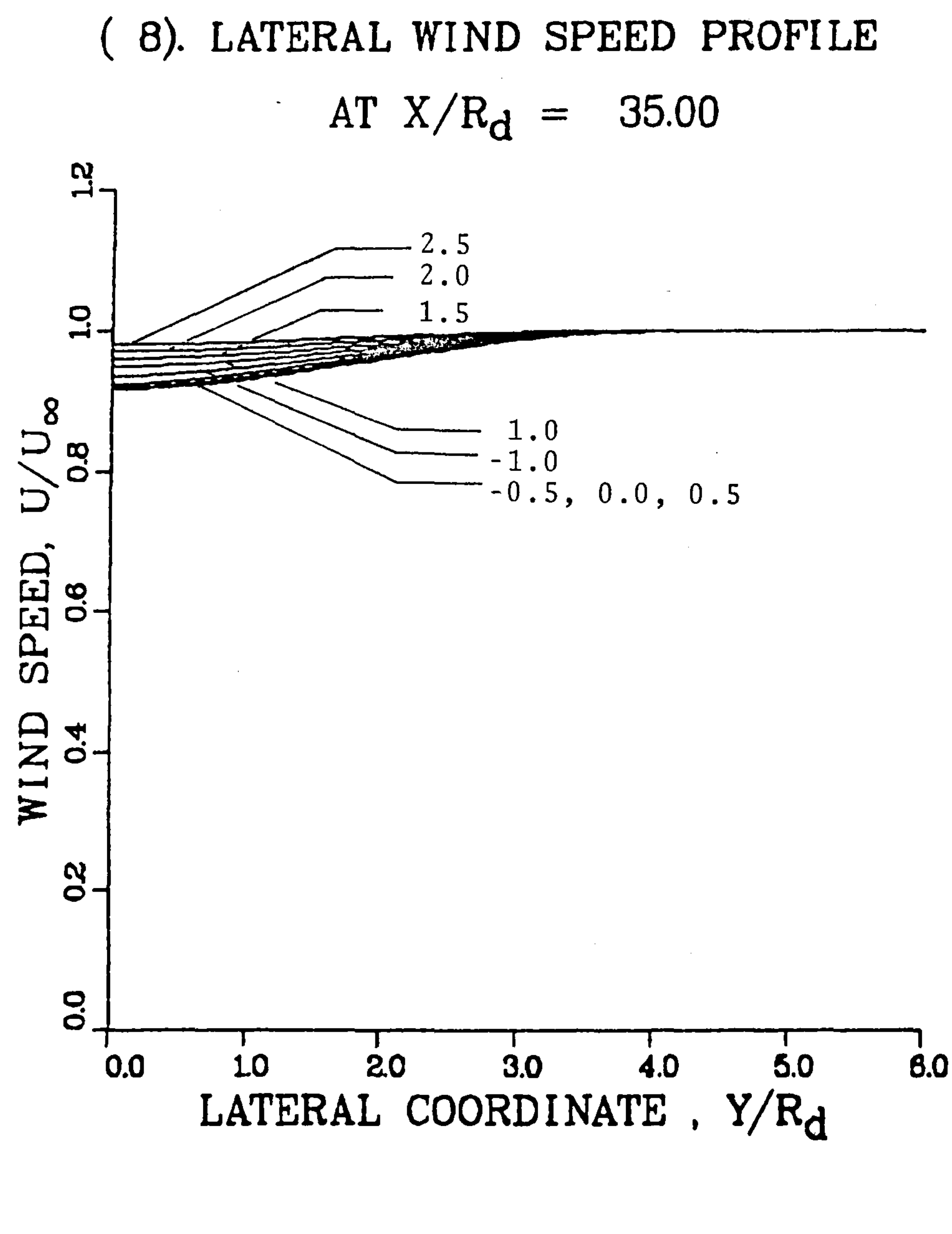


Figure 5-5. Wake plots for wake in ground effect for Pasquill stability class E (continued).

(9). LATERAL WIND SPEED PROFILE

AT X/R_d = 50.00

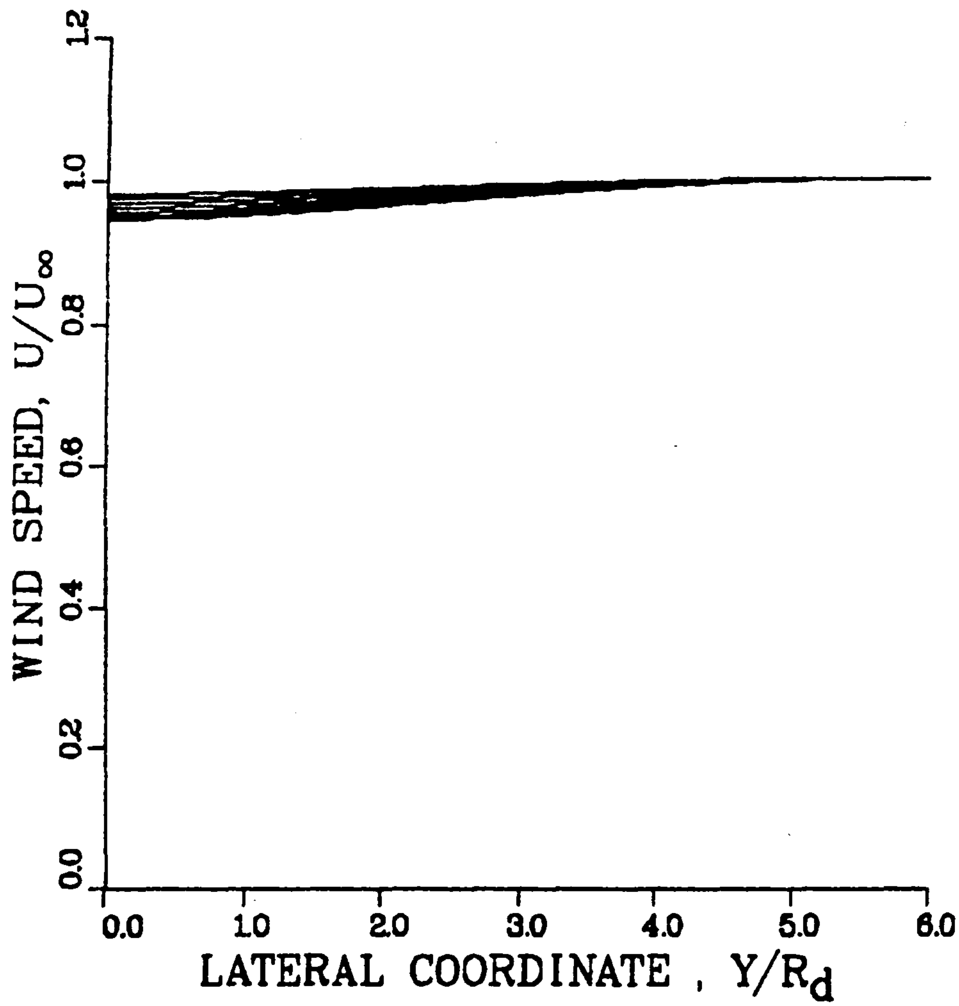


Figure 5-5. Wake plots for wake in ground effect for Pasquill stability class E (continued).

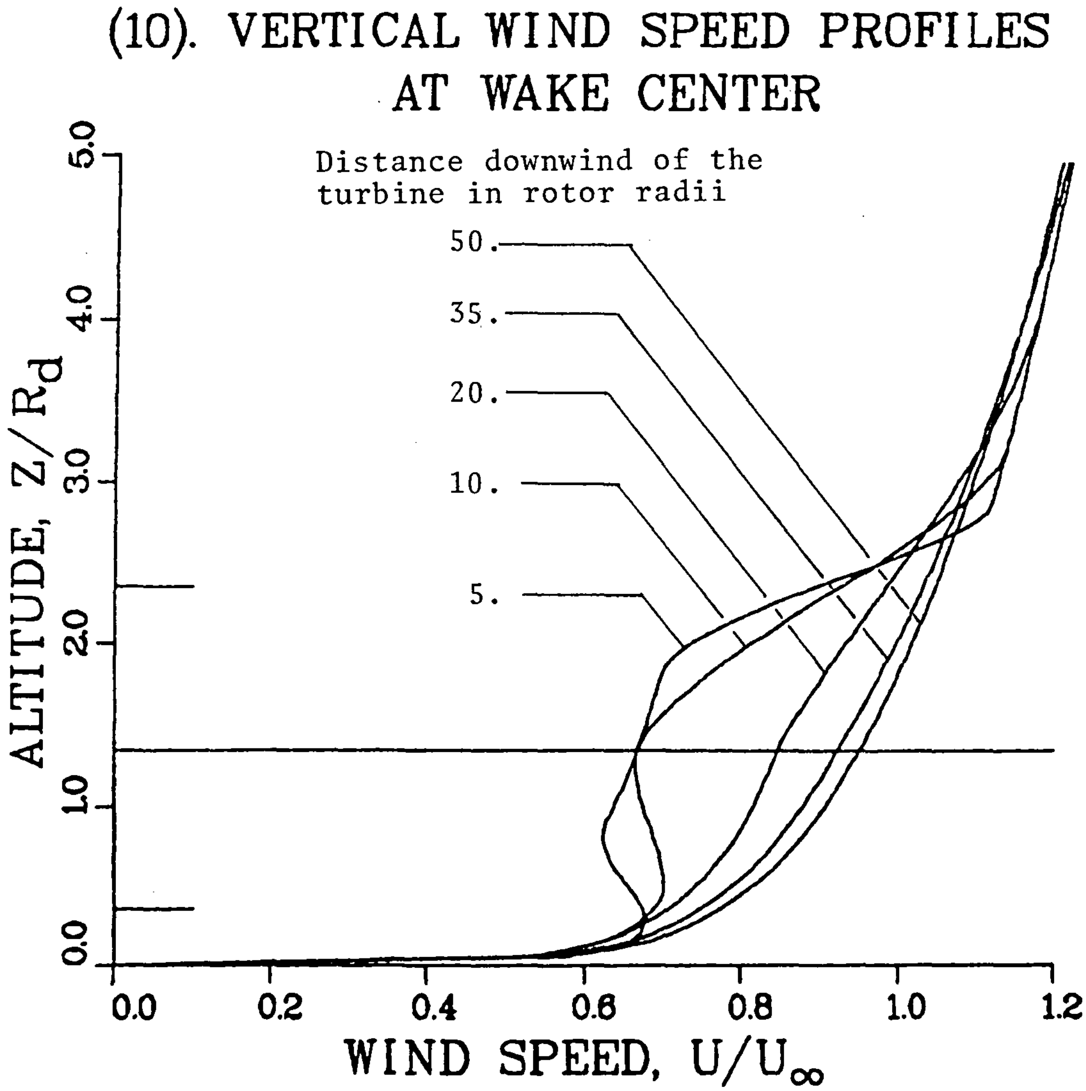


Figure 5-5. Wake plots for wake in ground effect for Pasquill stability class E (concluded).

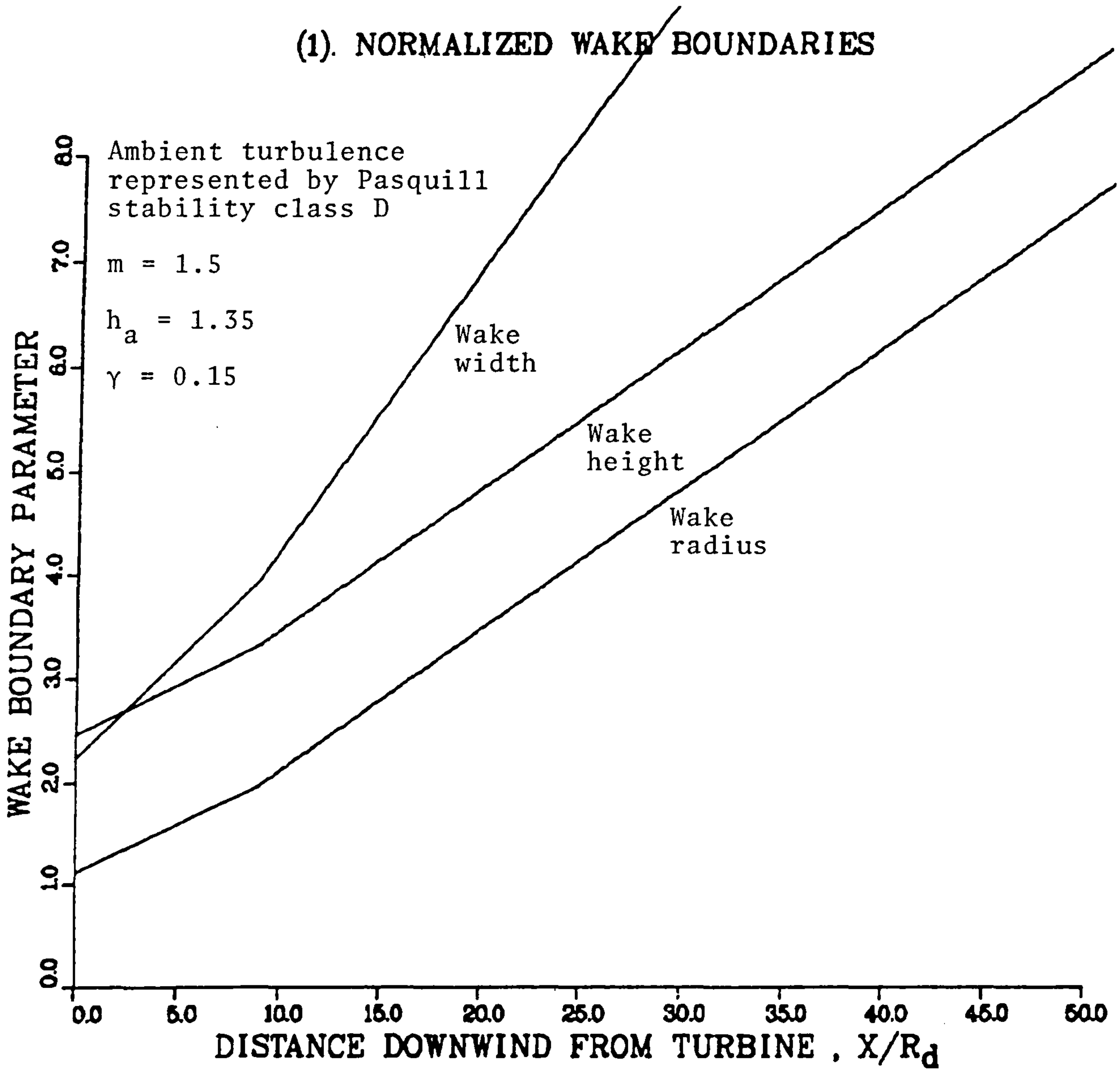


Figure 5-6. Wake plots for wake in ground effect for Pasquill stability class D.

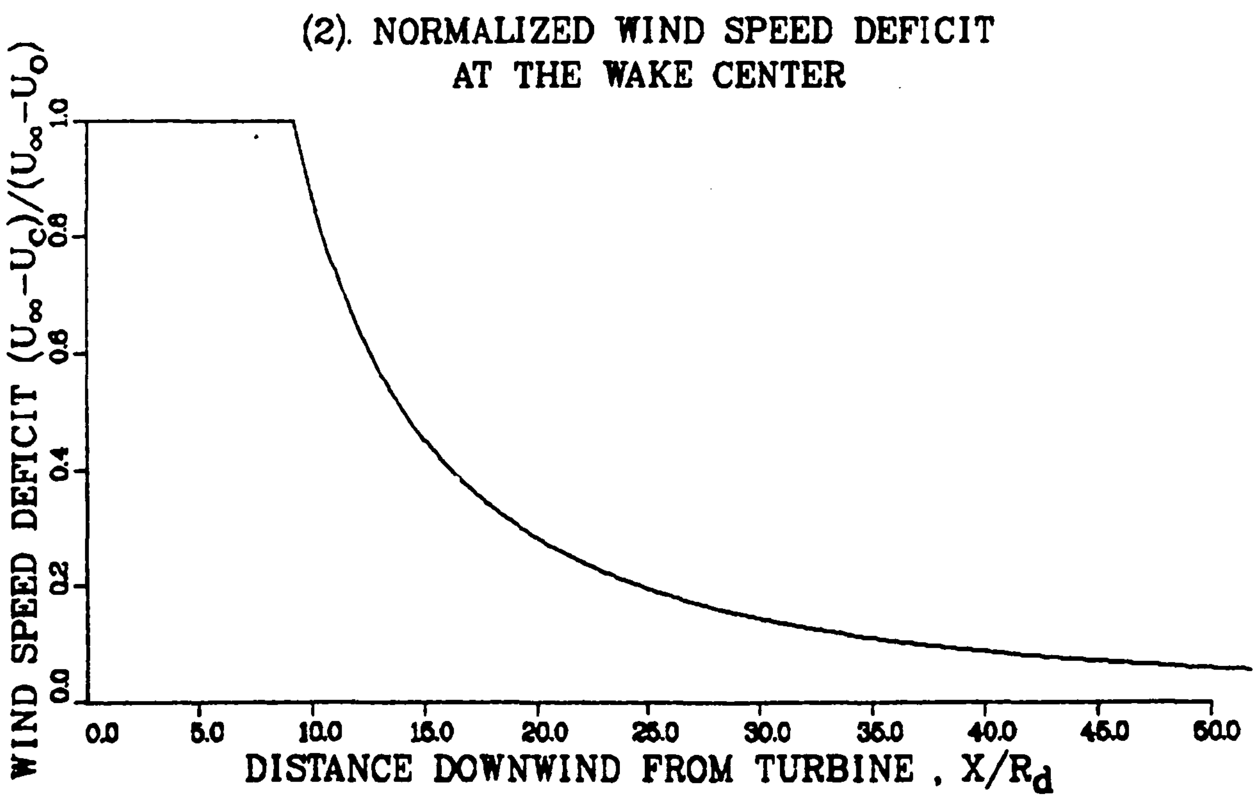


Figure 5-6. Wake plots for wake in ground effect for Pasquill stability class D (continued).

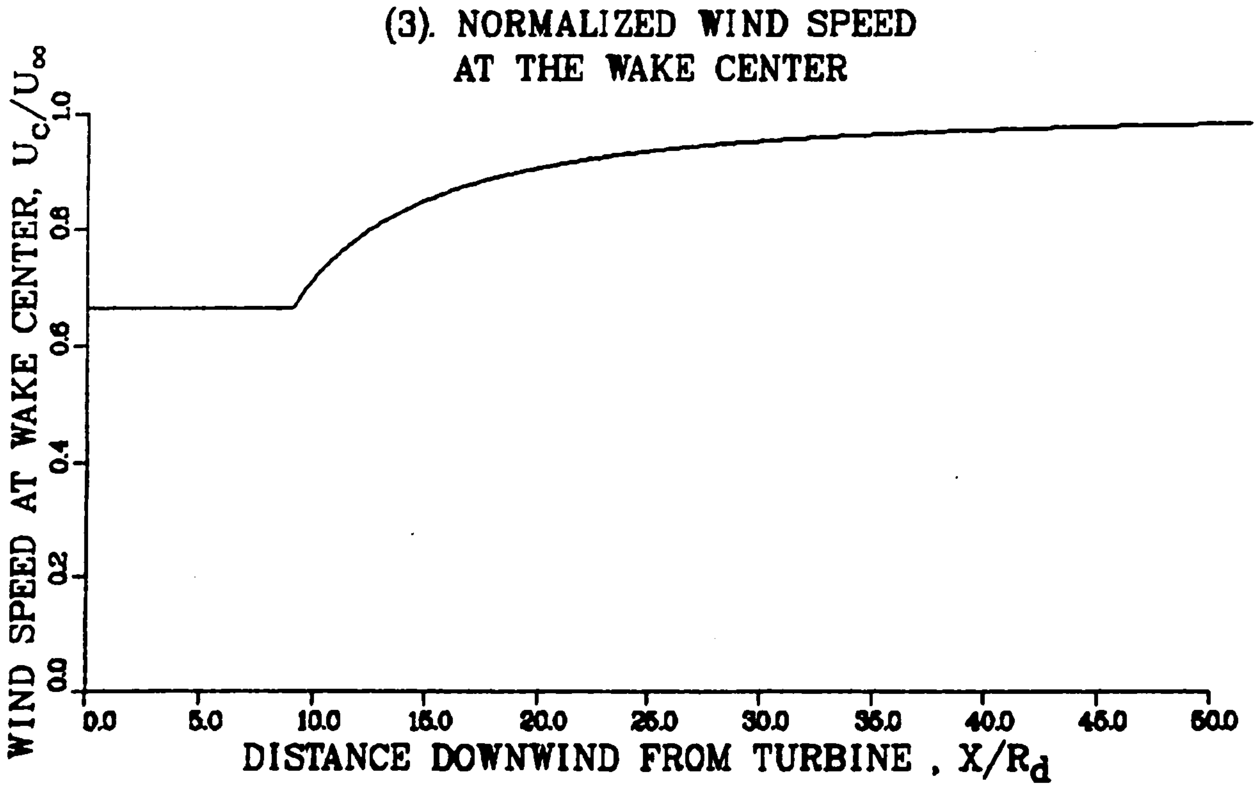


Figure 5-6. Wake plots for wake in ground effect for Pasquill stability class D (continued).

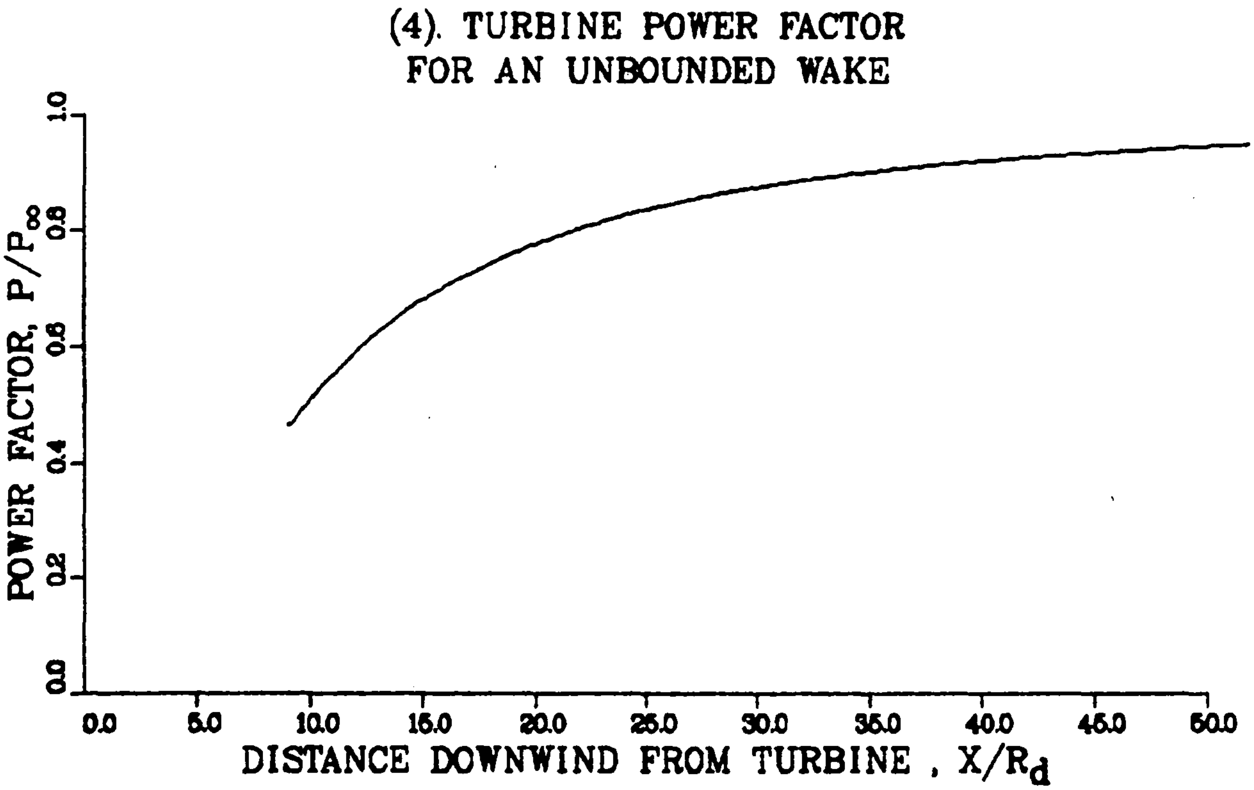


Figure 5-6. Wake plots for wake in ground effect for Pasquill stability class D (continued).

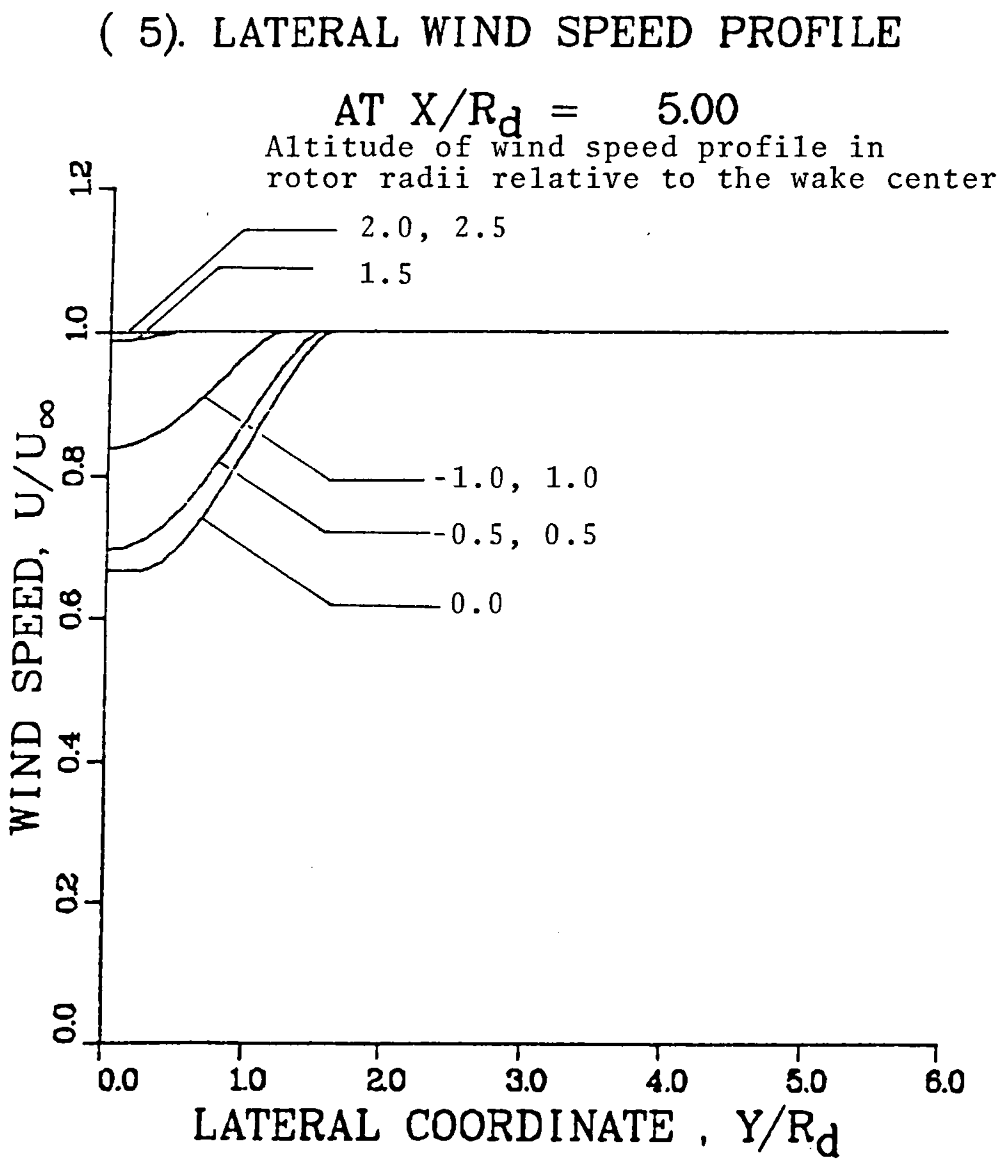


Figure 5-6. Wake plots for wake in ground effect for Pasquill stability class D (continued).

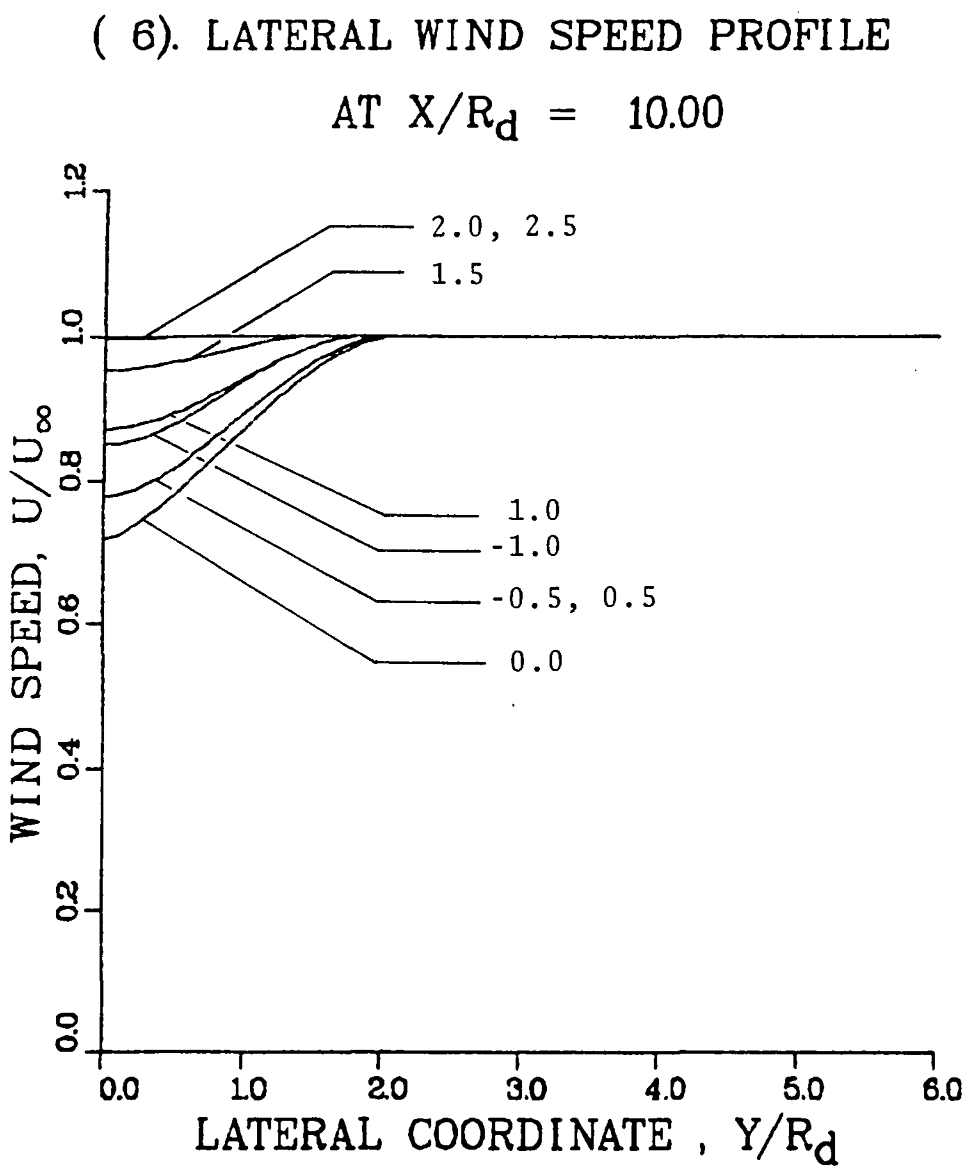


Figure 5-6. Wake plots for wake in ground effect for Pasquill stability class D (continued).

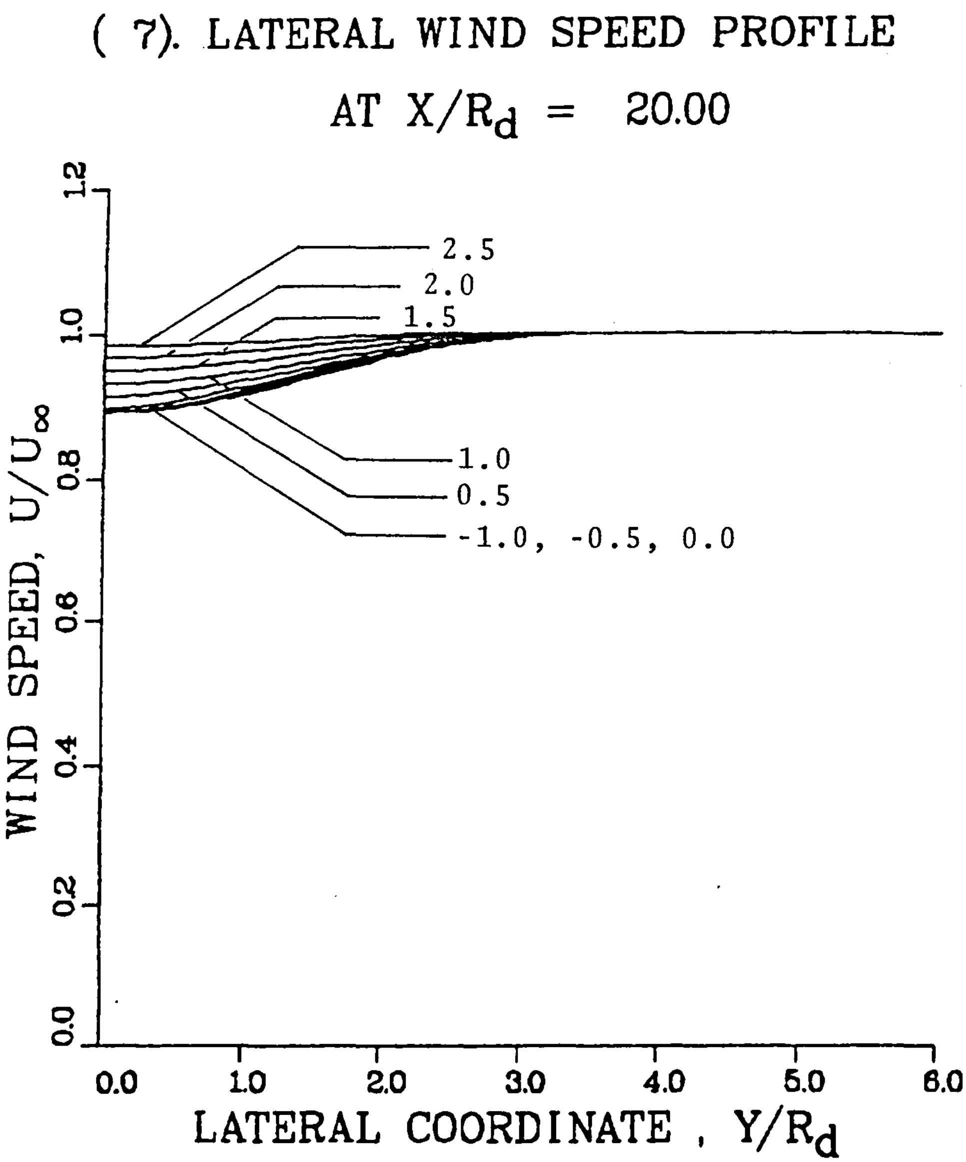


Figure 5-6. Wake plots for wake in ground effect for Pasquill stability class D (continued).

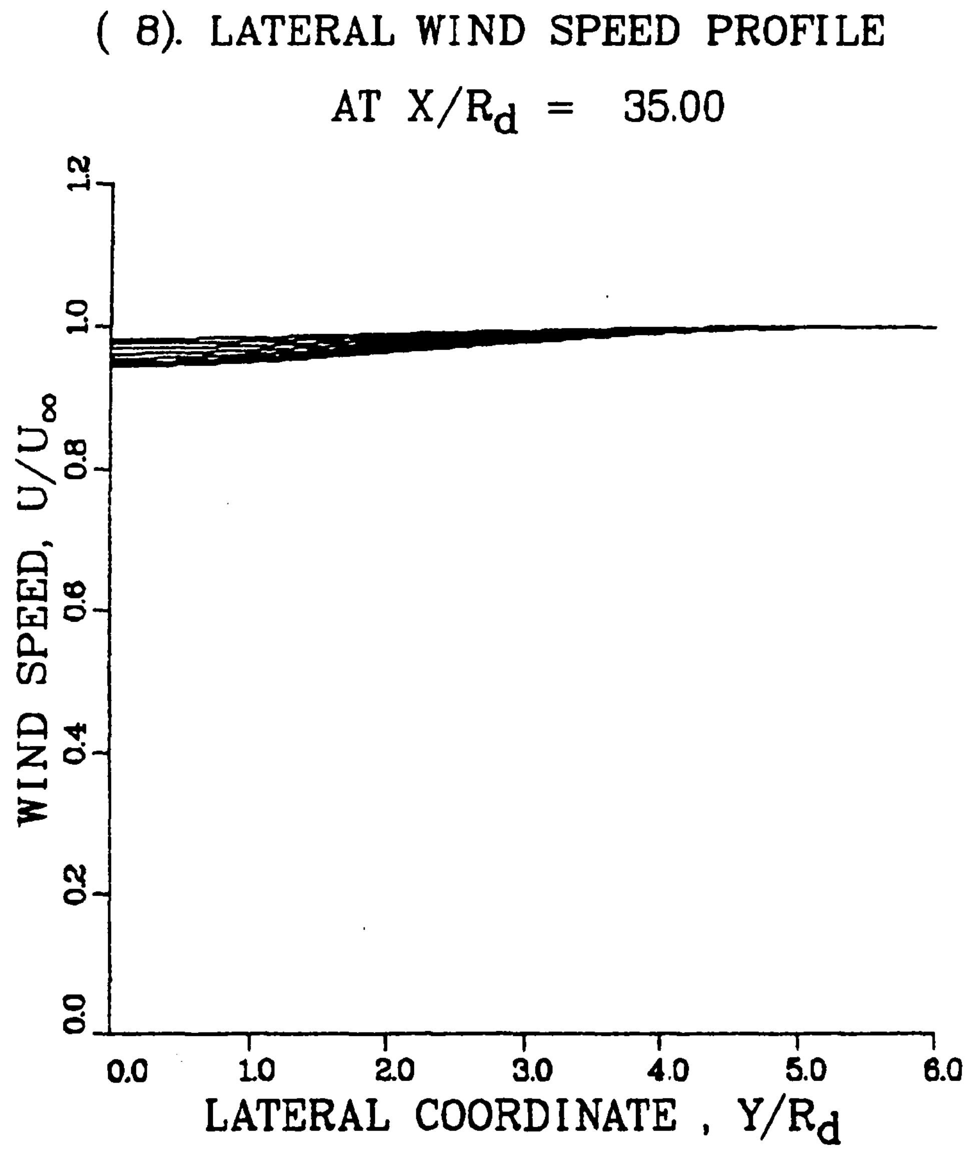


Figure 5-6. Wake plots for wake in ground effect for Pasquill stability class D (continued).

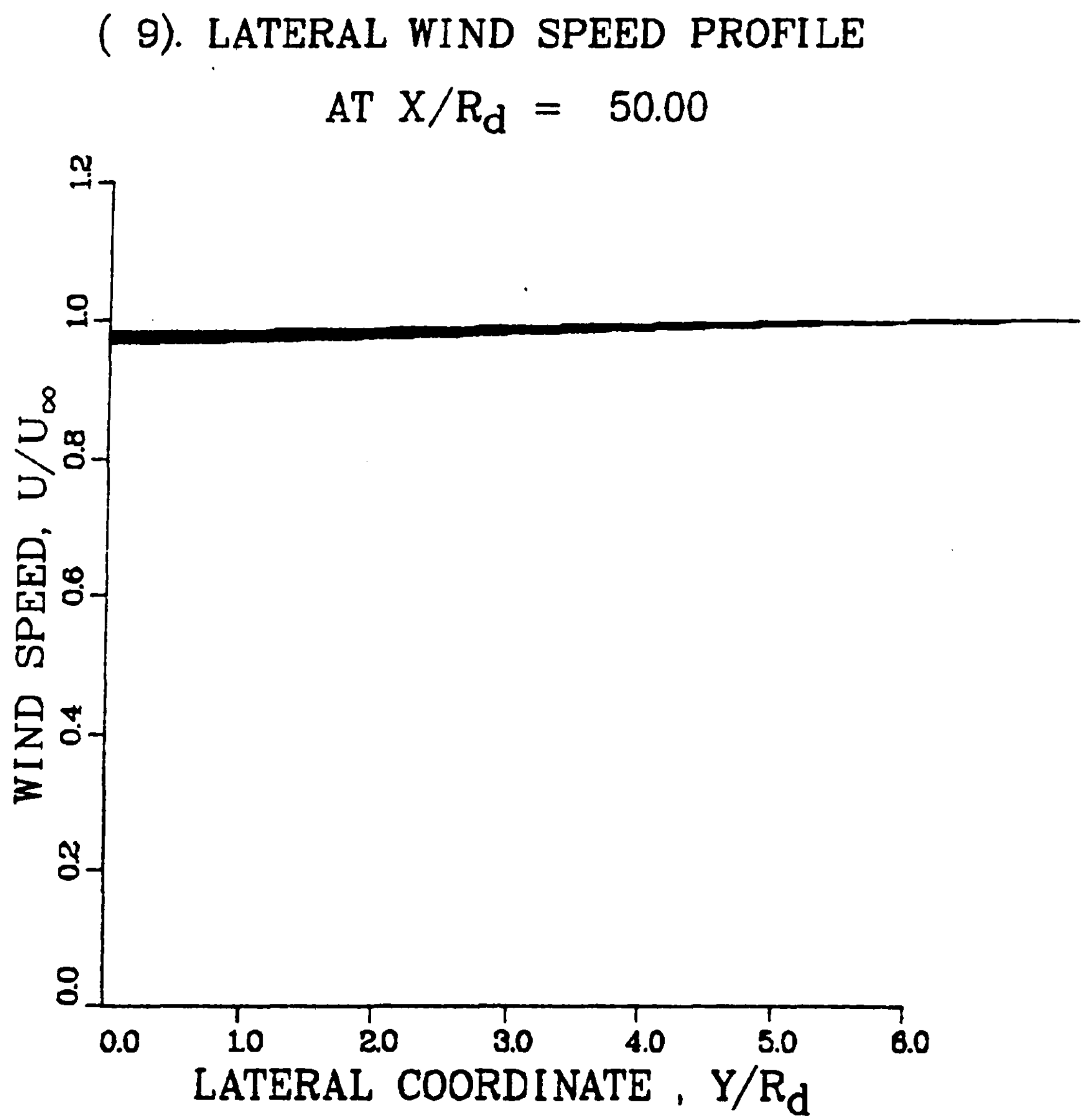


Figure 5-6. Wake plots for wake in ground effect for Pasquill stability class D (continued).

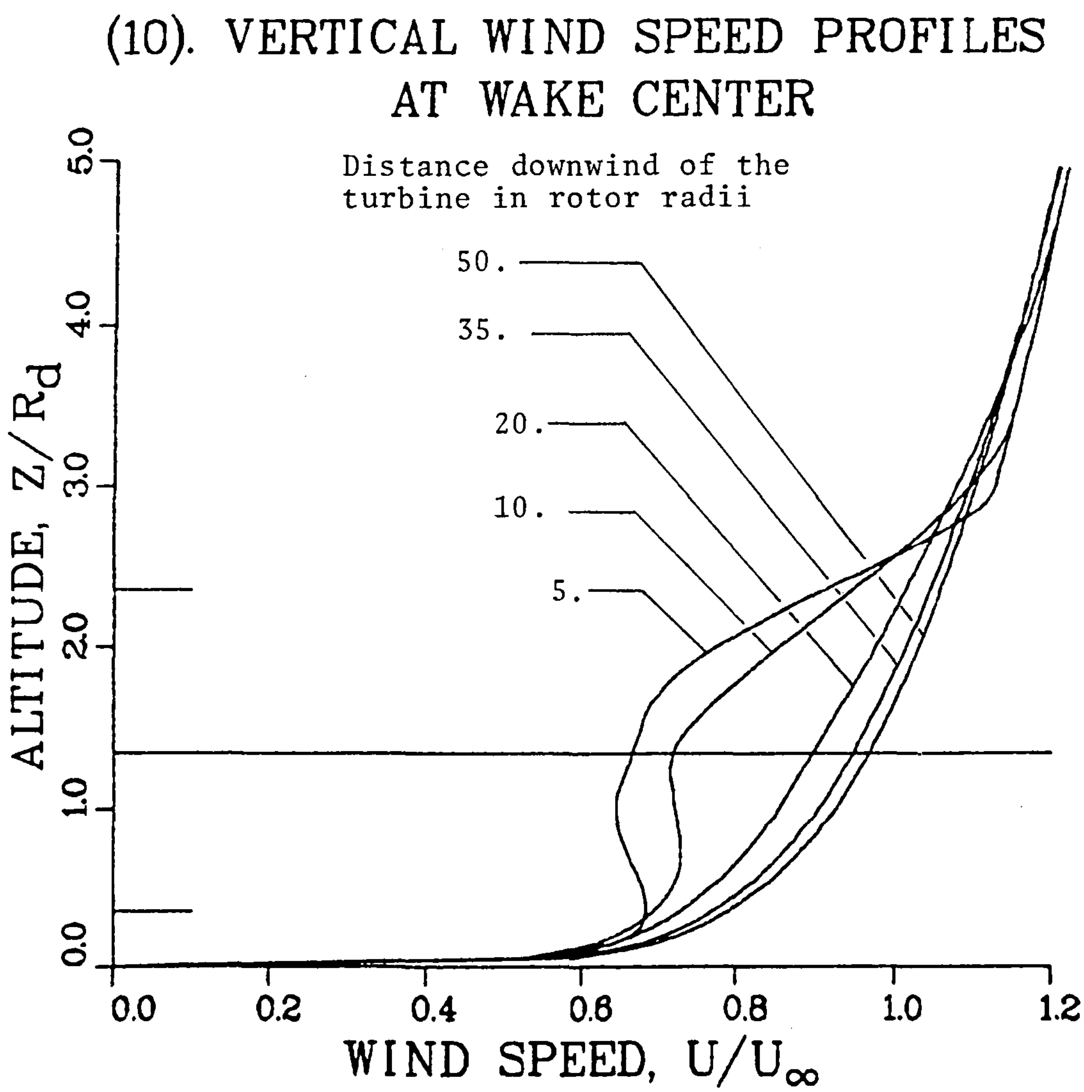


Figure 5-6. Wake plots for wake in ground effect for Pasquill stability class D (concluded).

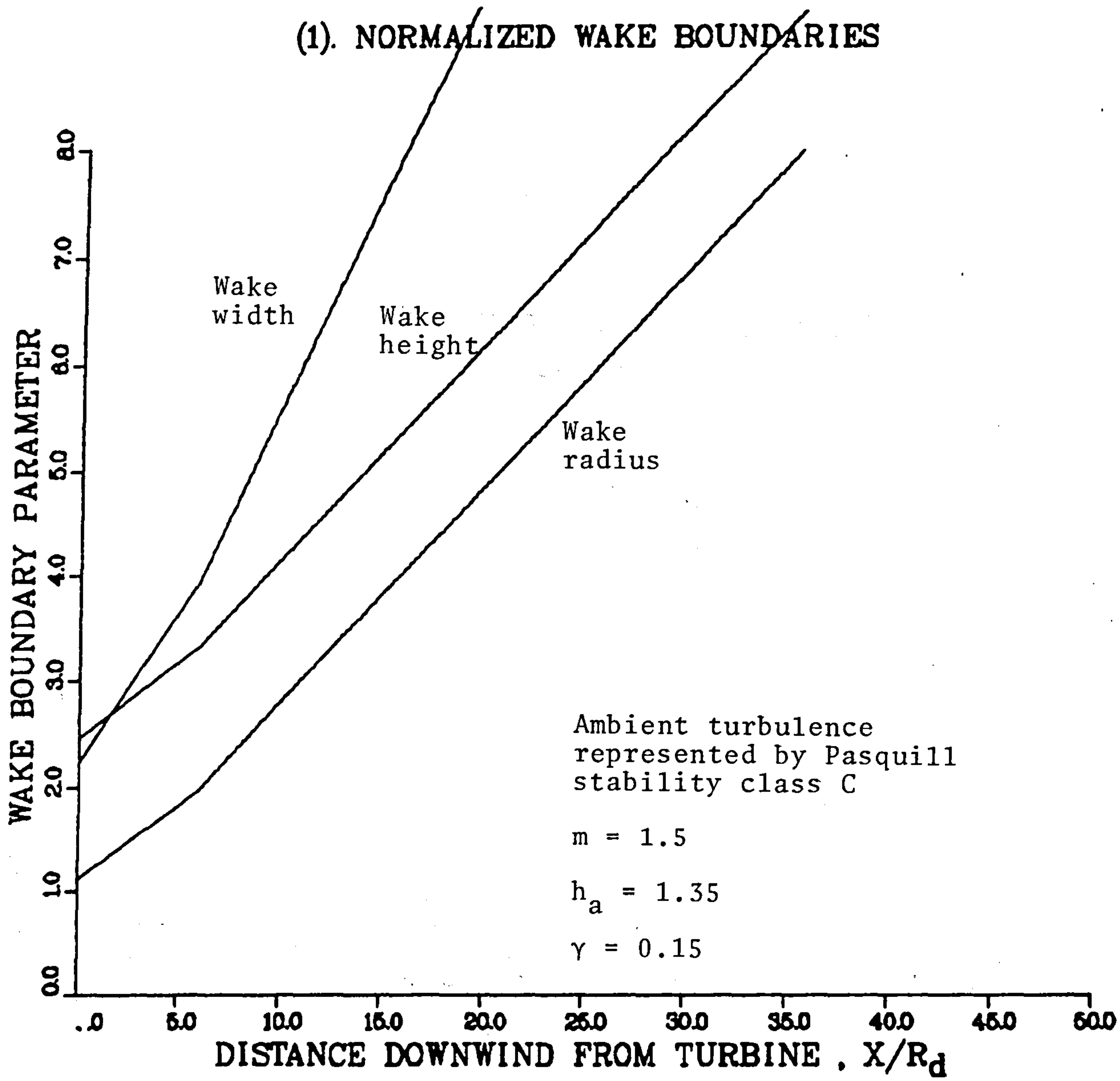
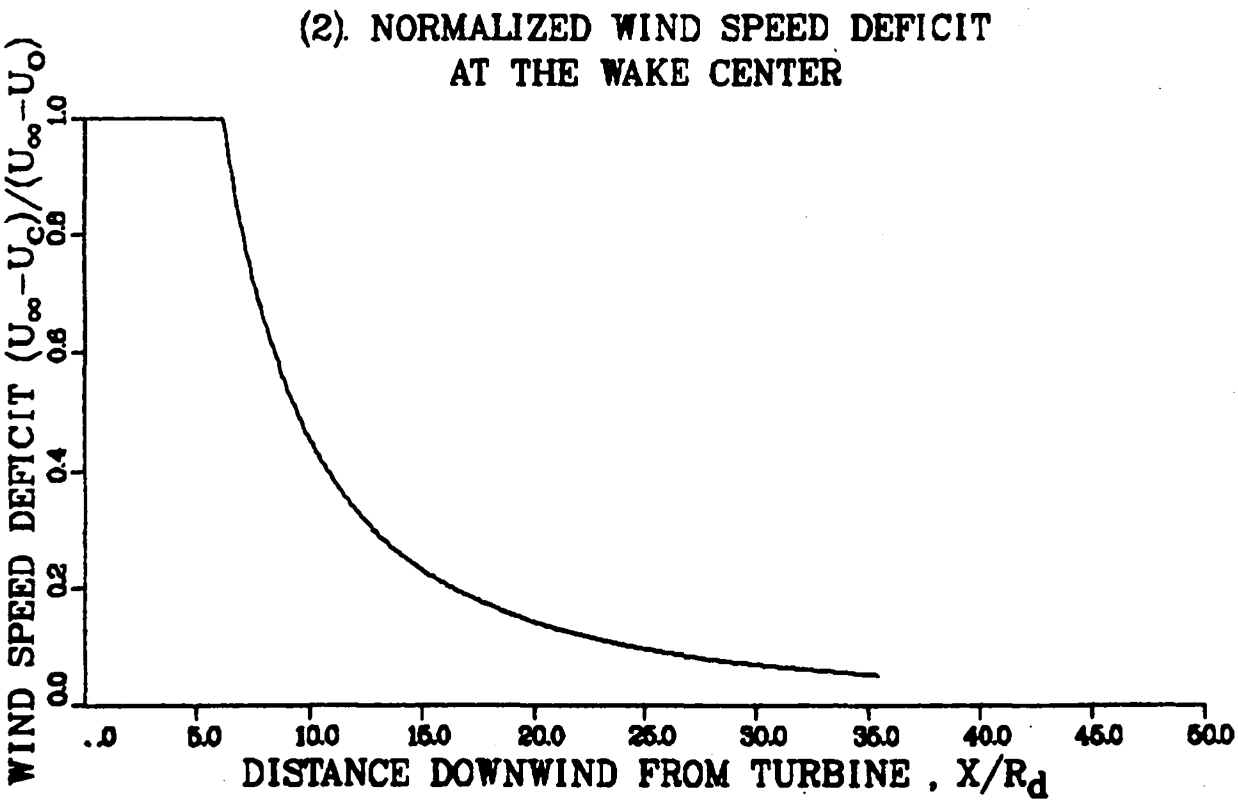


Figure 5-7. Wake plots for wake in ground effect for Pasquill stability class C.



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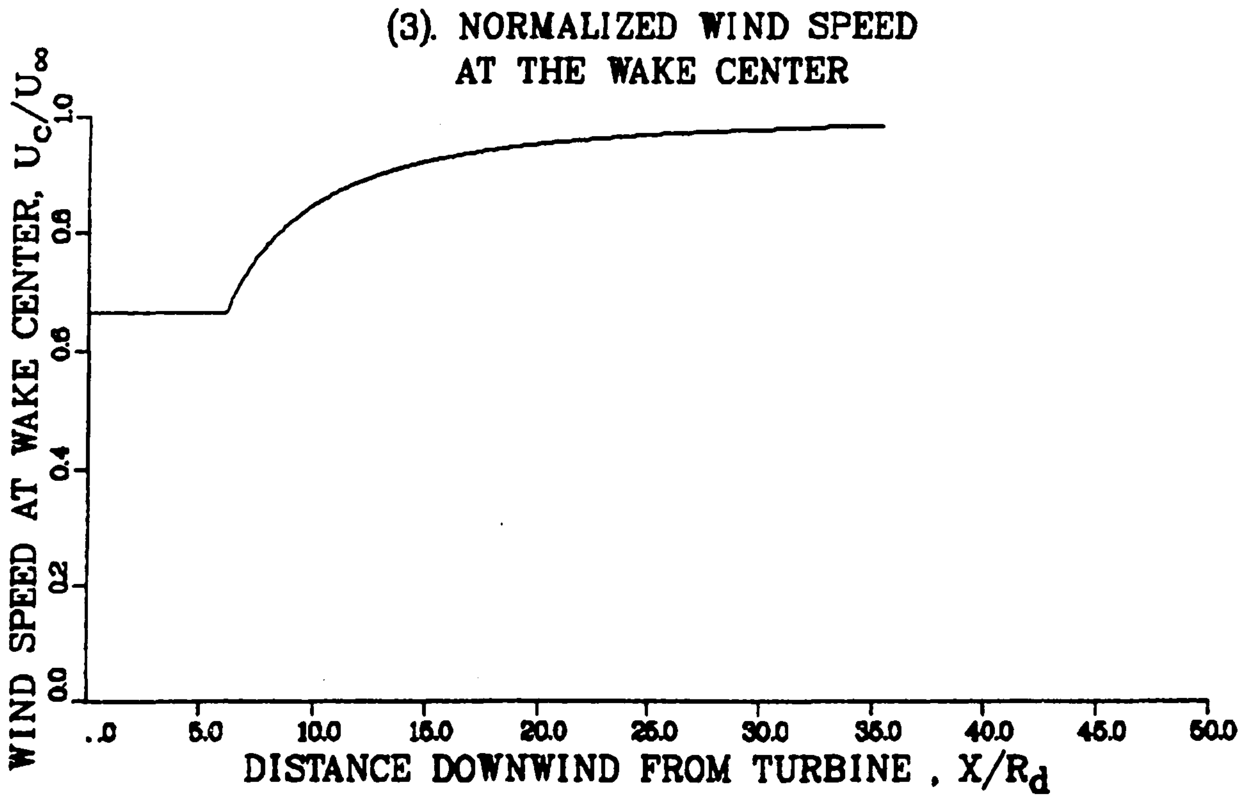

Figure 5-7. Wake plots for wake in ground effect for Pasquill stability class C (continued).

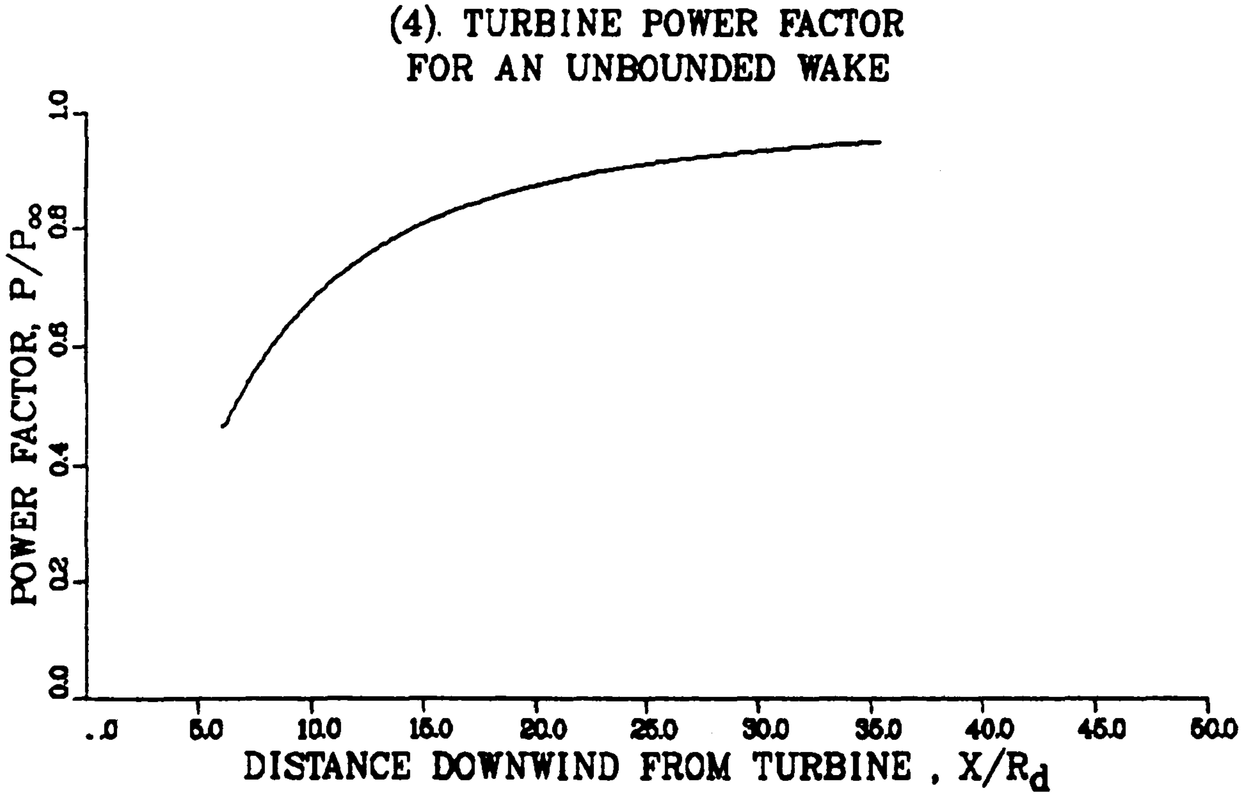

Figure 5-7. Wake plots for wake in ground effect for Pasquill stability class C (continued).

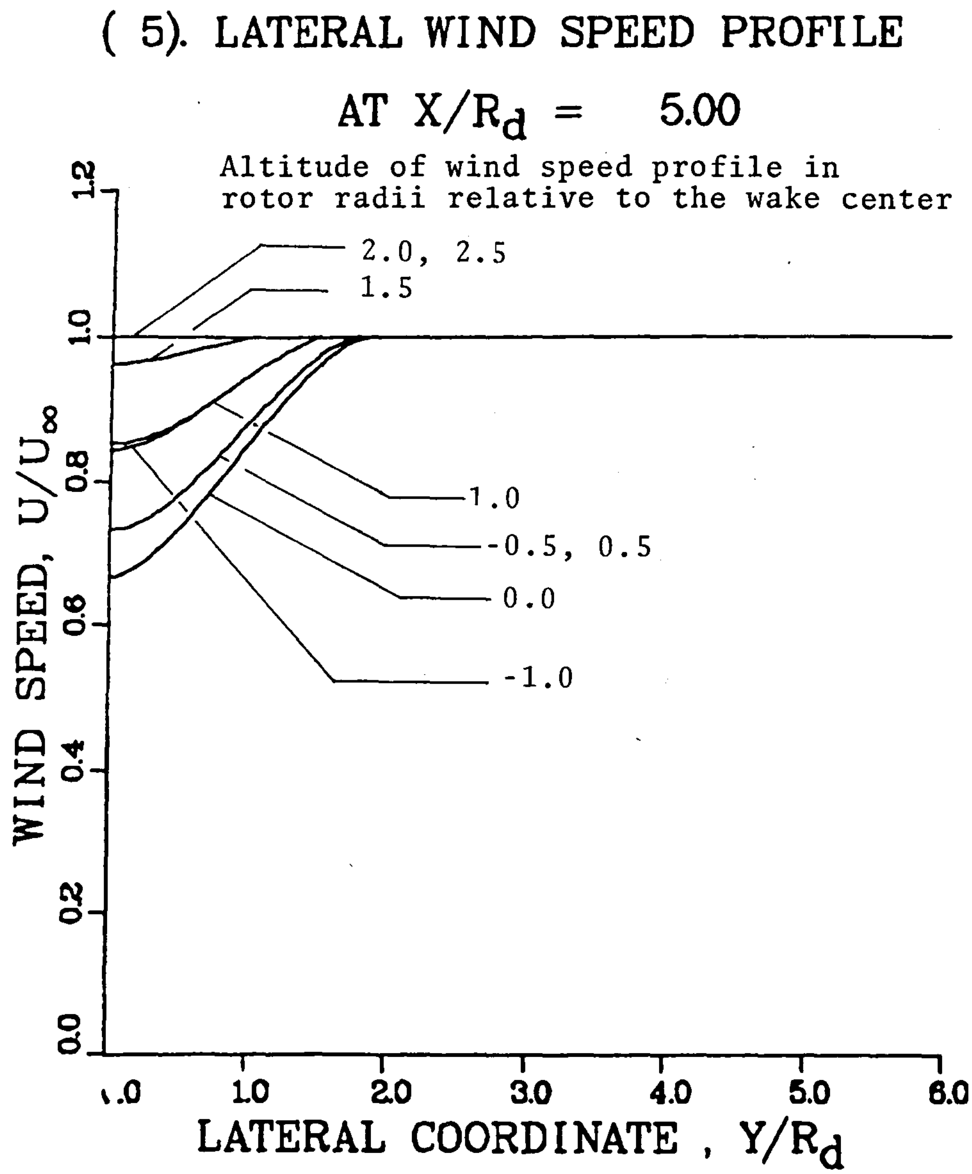

Figure 5-7. Wake plots for wake in ground effect for Pasquill stability class C (continued).

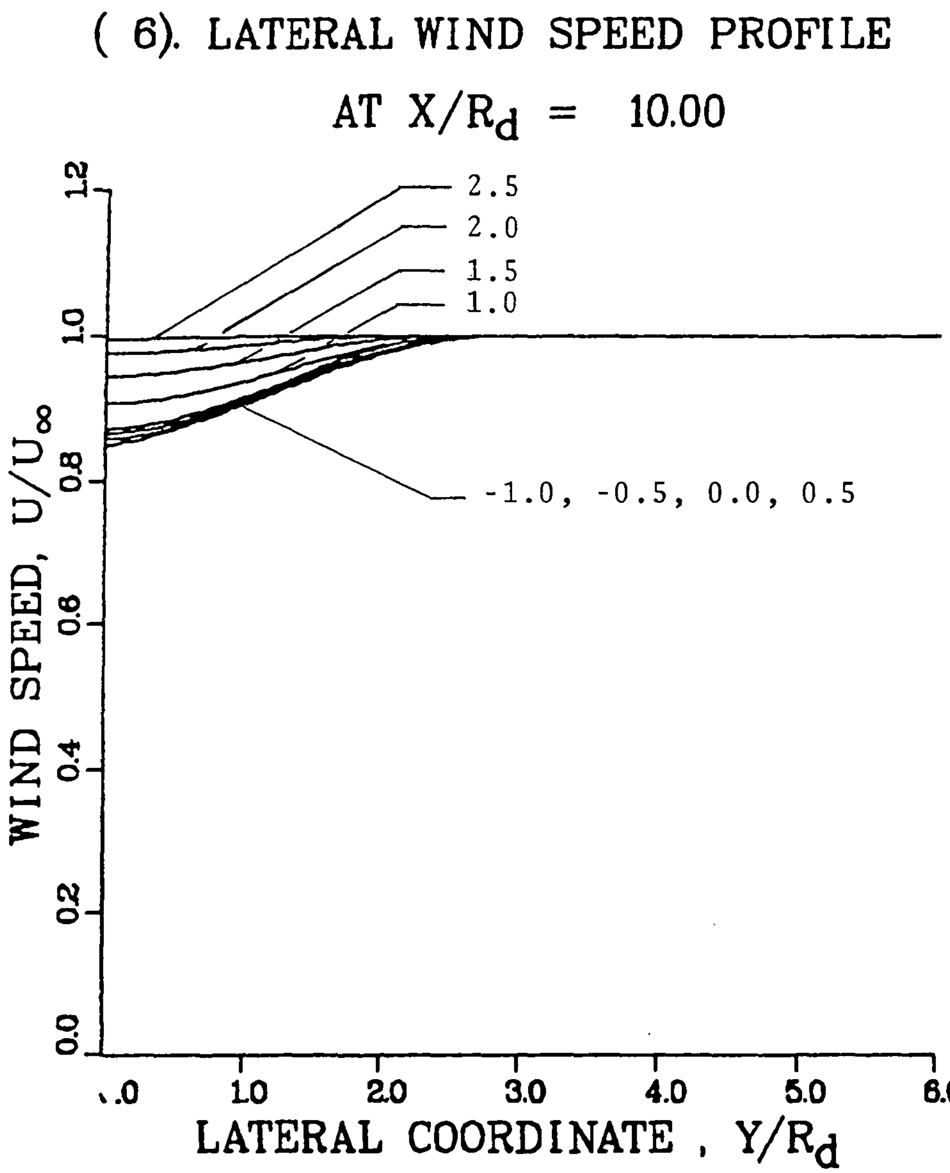

Figure 5-7. Wake plots for wake in ground effect for Pasquill stability class C (continued).

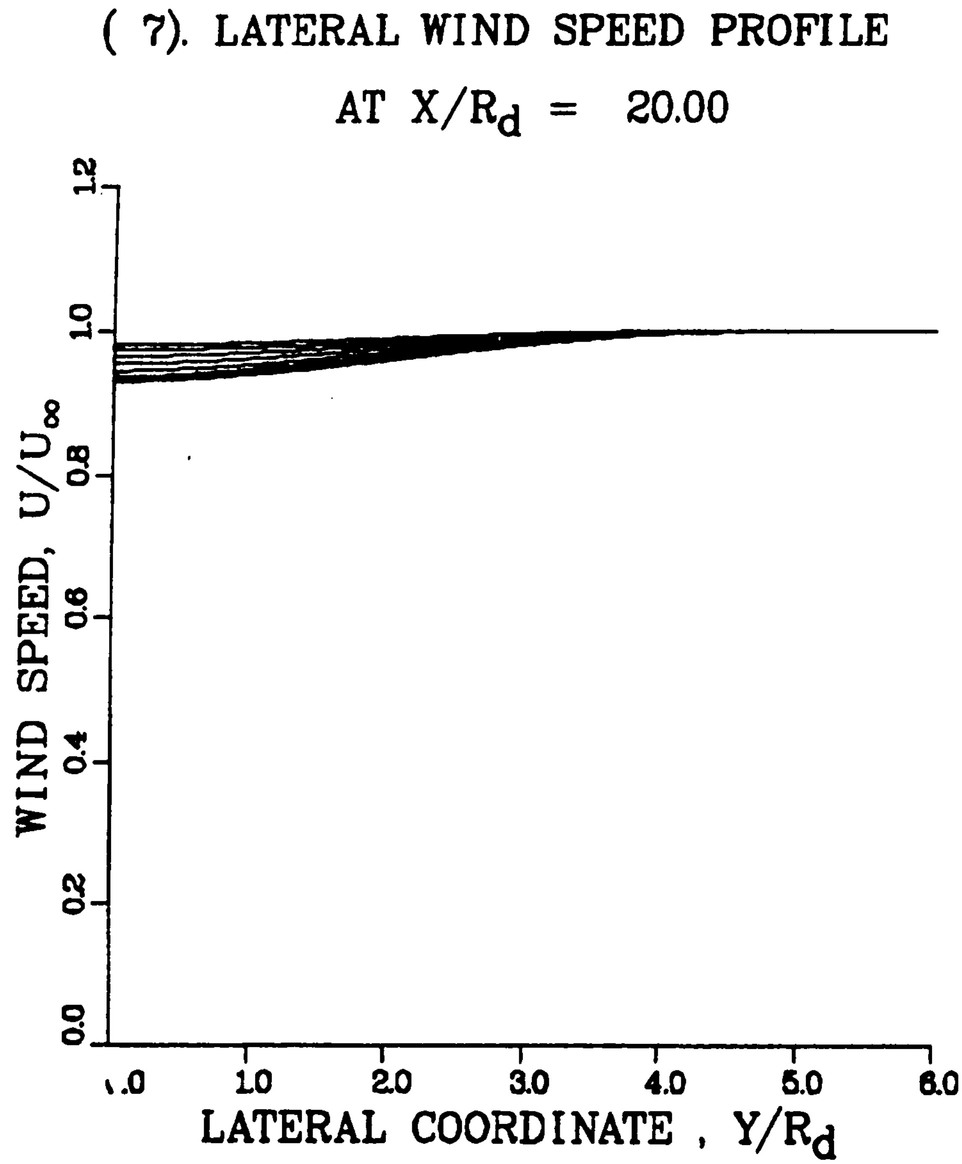

Figure 5-7. Wake plots for wake in ground effect for Pasquill stability class C (continued).

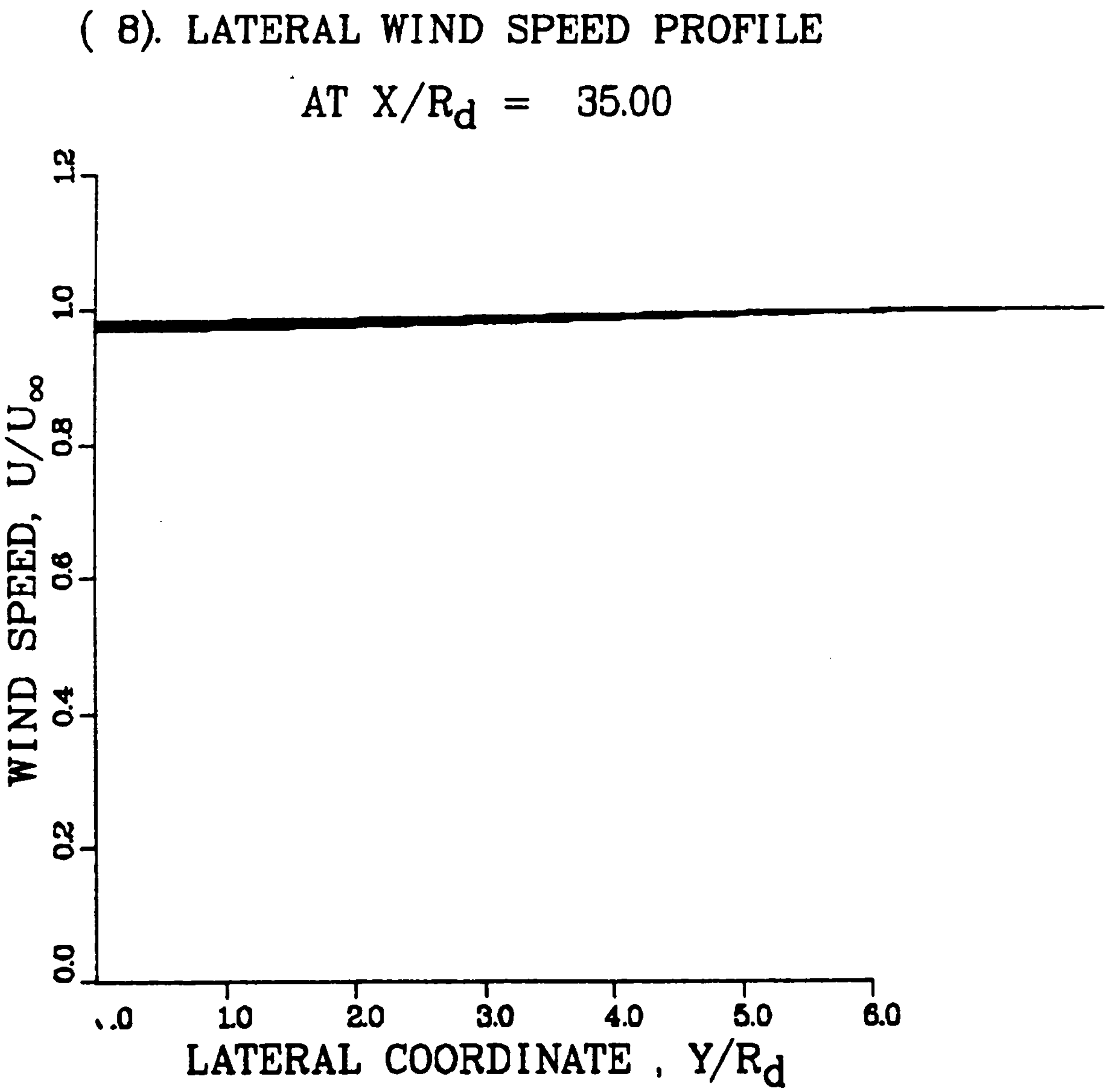

Figure 5-7. Wake plots for wake in ground effect for Pasquill stability class C (continued).

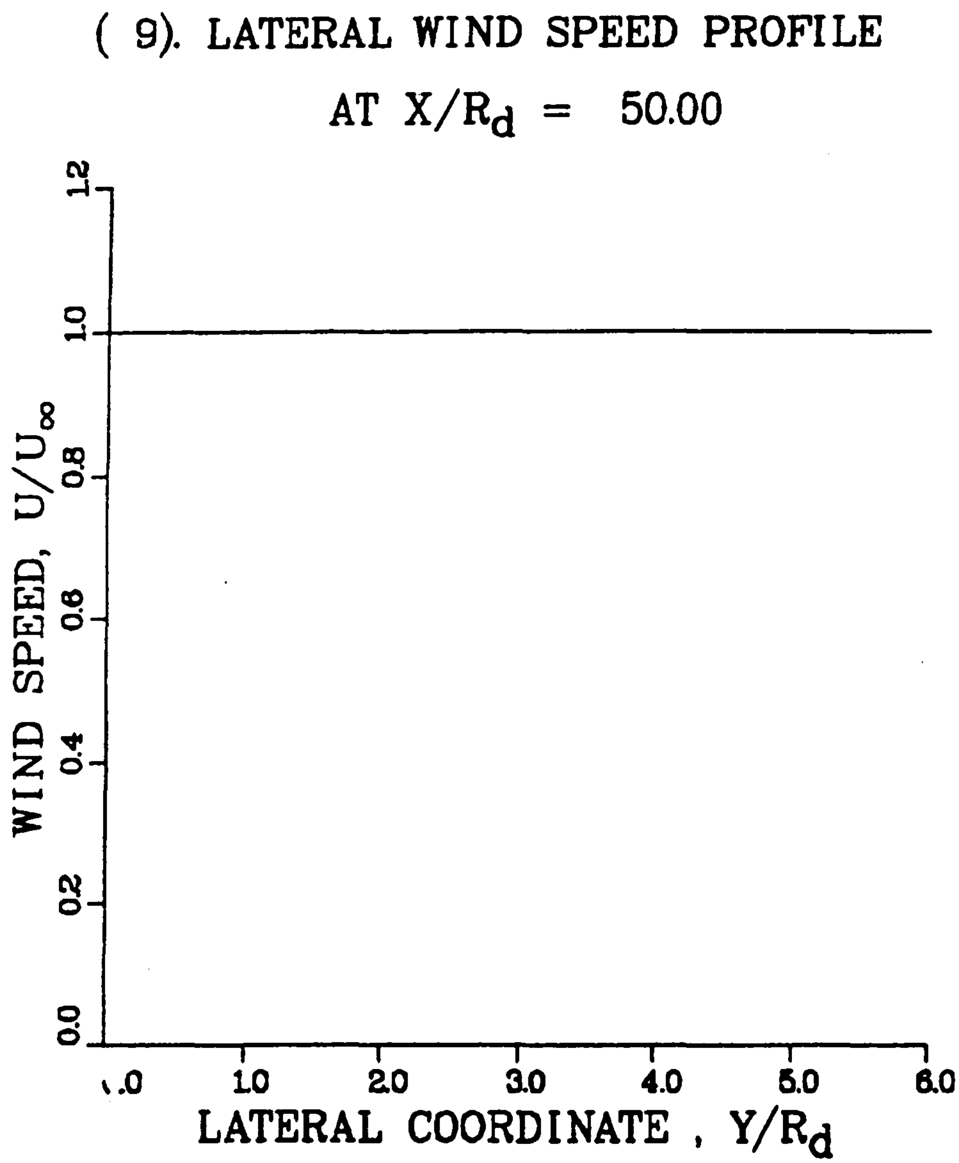

Figure 5-7. Wake plots for wake in ground effect for Pasquill stability class C (continued).

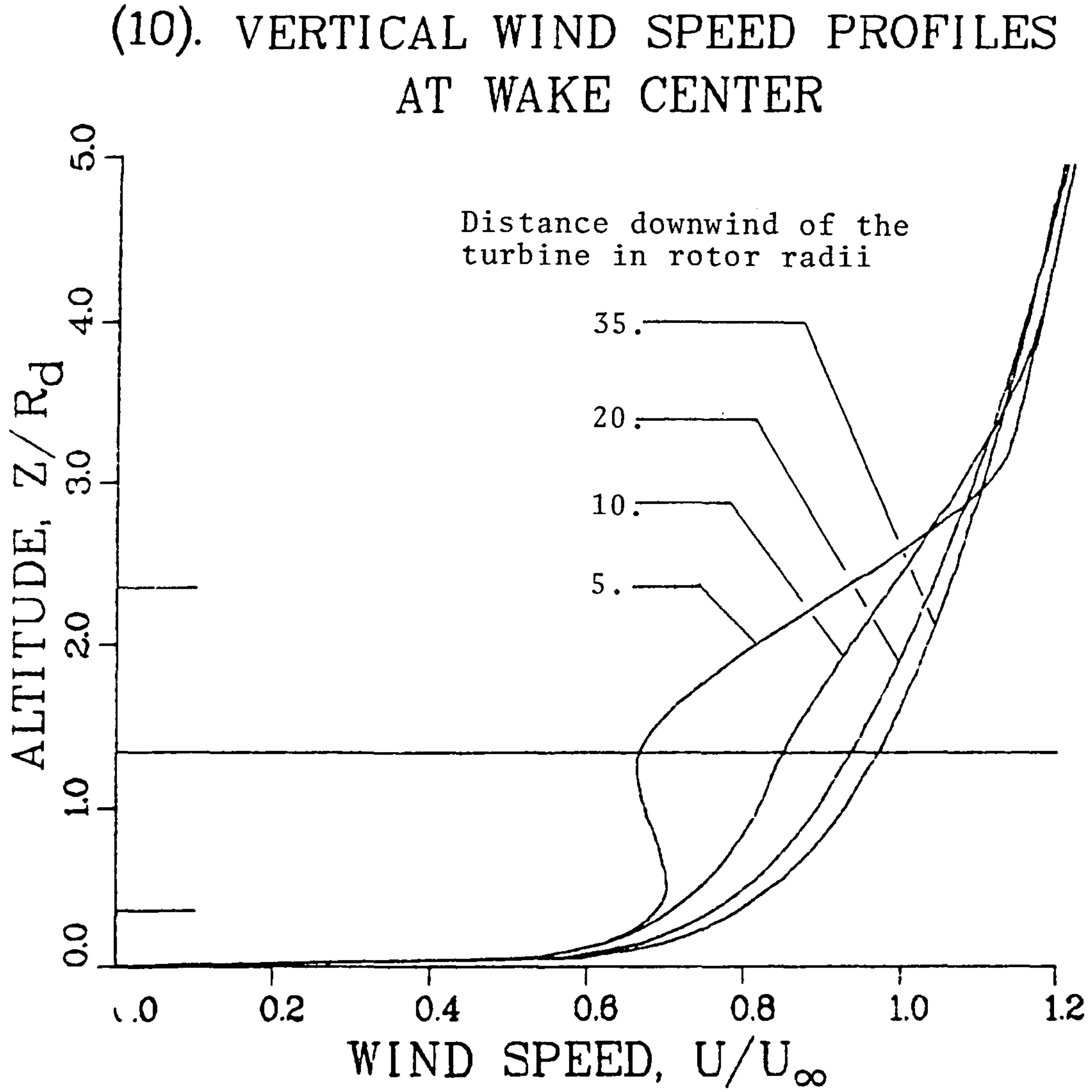

Figure 5-7. Wake plots for wake in ground effect for Pasquill stability class C (concluded).

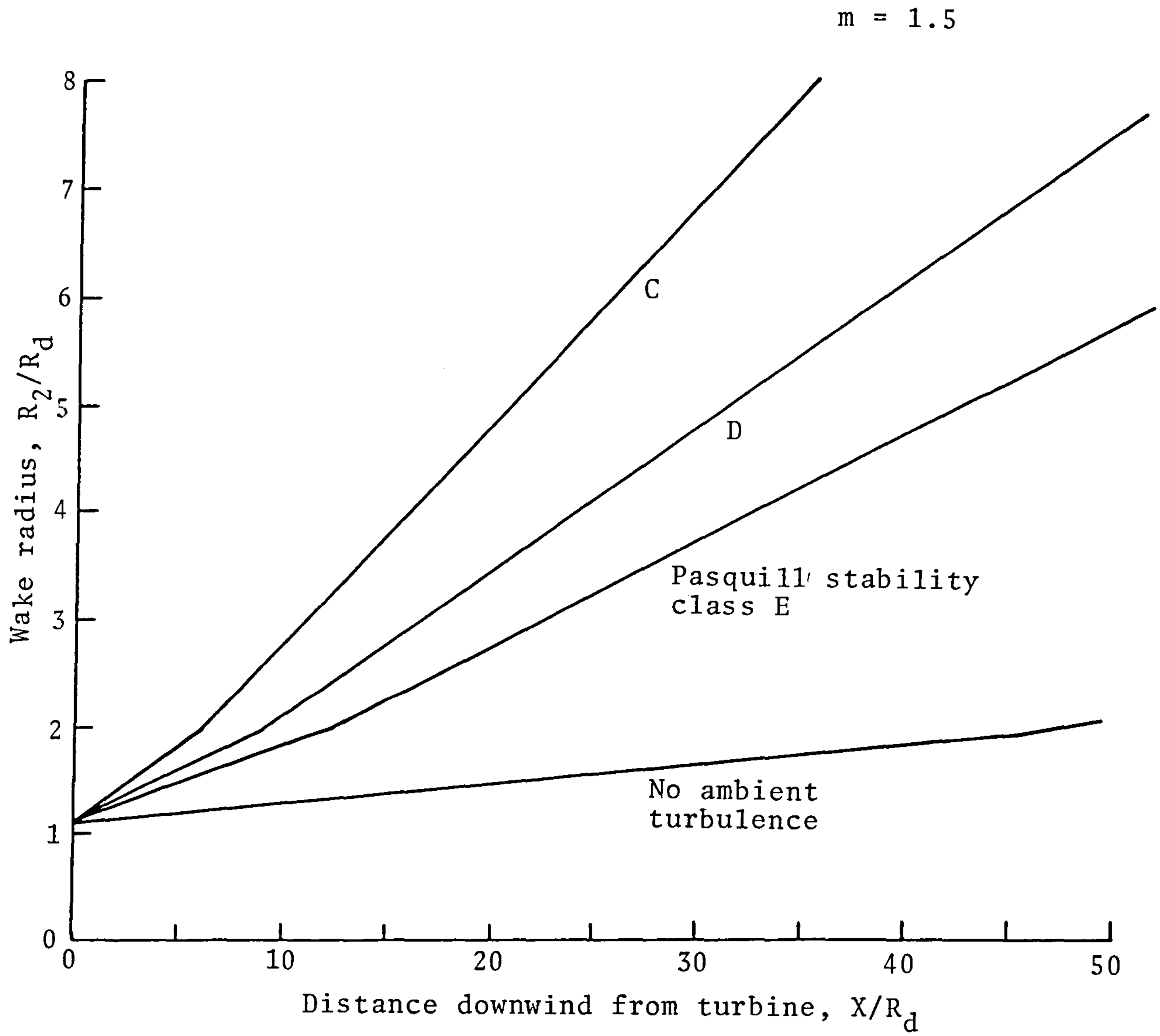

Figure 5-8. Effect of ambient turbulence on wake radius.

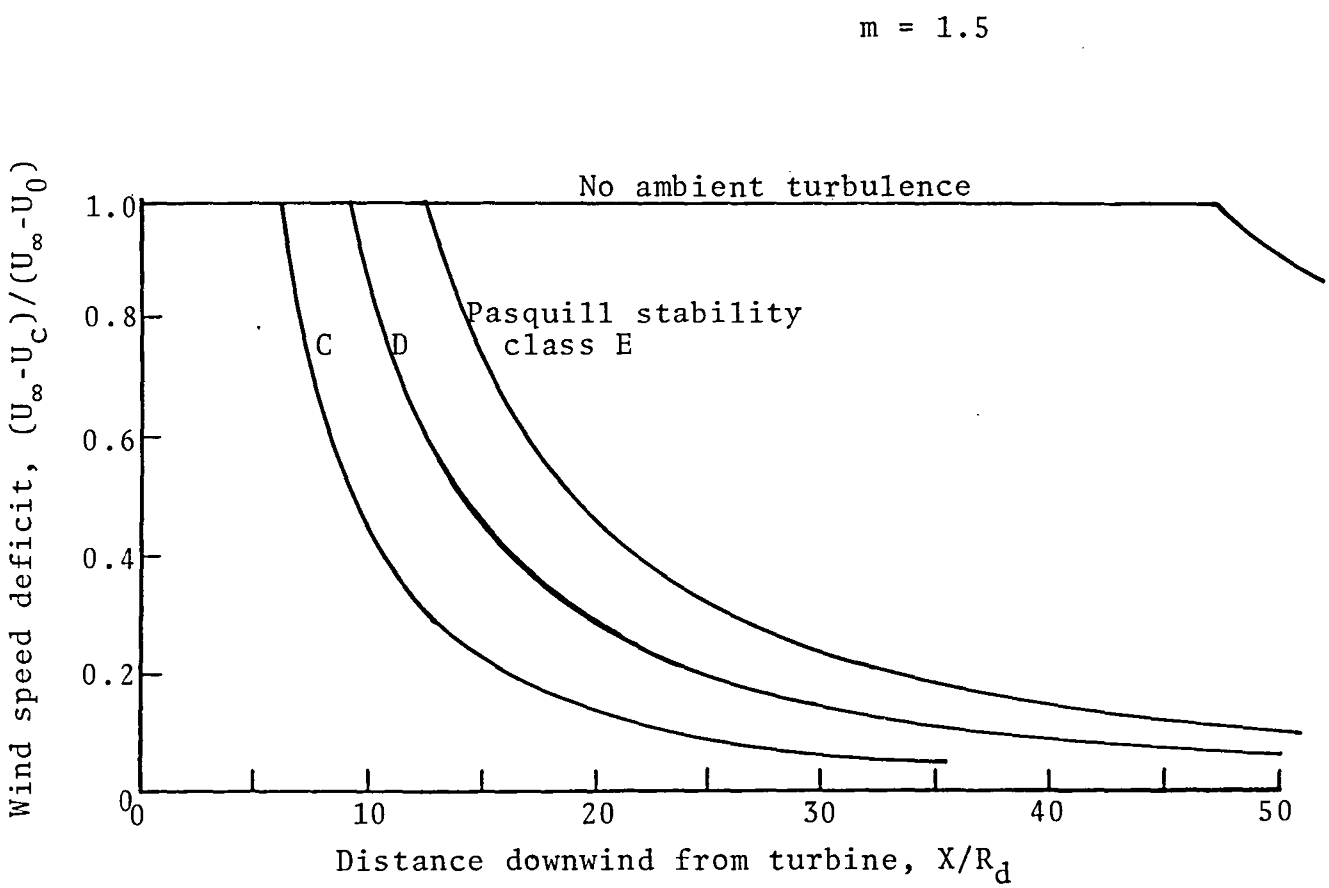

Figure 5-9. Effect of ambient turbulence on wind speed deficit parameter.

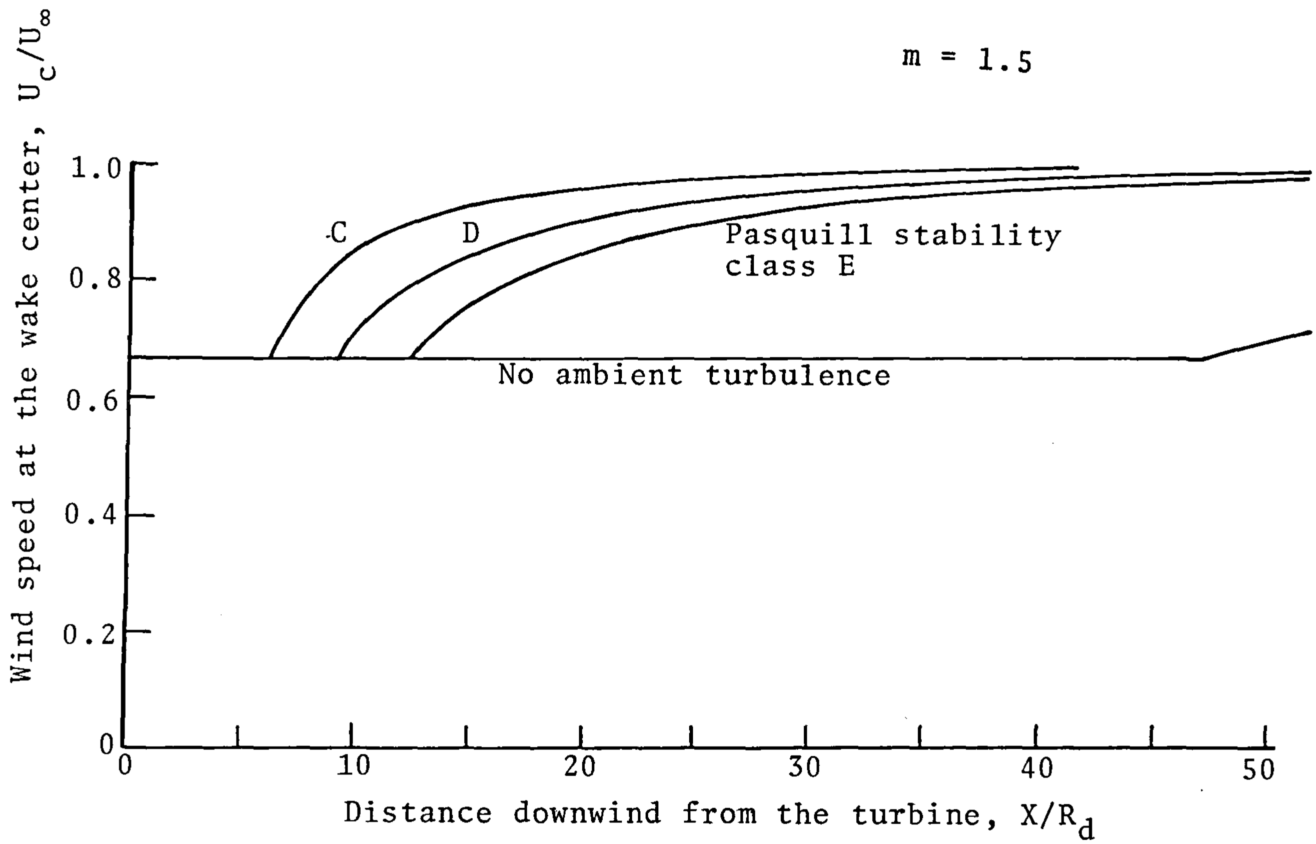

Figure 5-10. Effect of ambient turbulence on wind speed at the wake center.

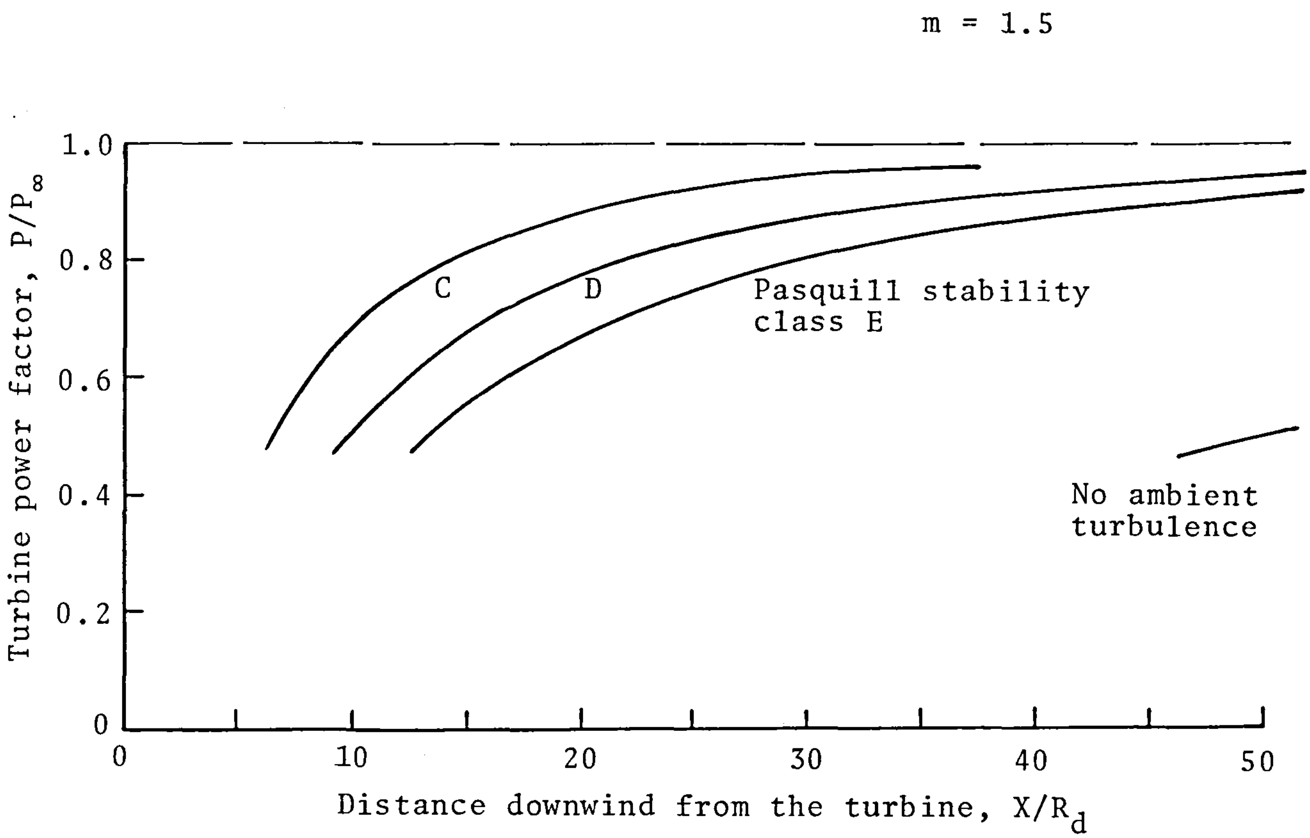

Figure 5-11. Effect of ambient turbulence on turbine power factor.

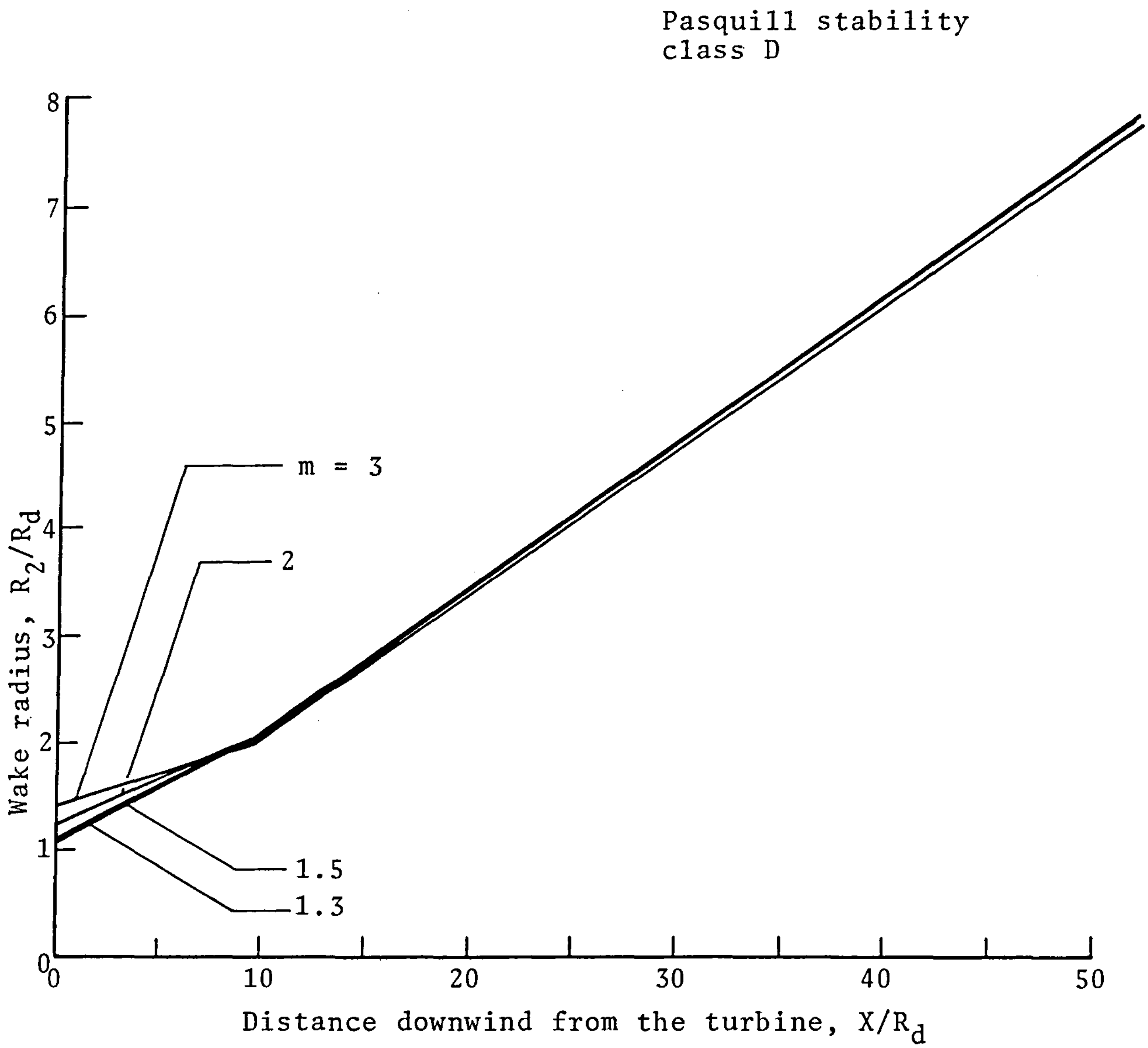

Figure 5-12. Effect of initial velocity ratio in the wake on wake radius.

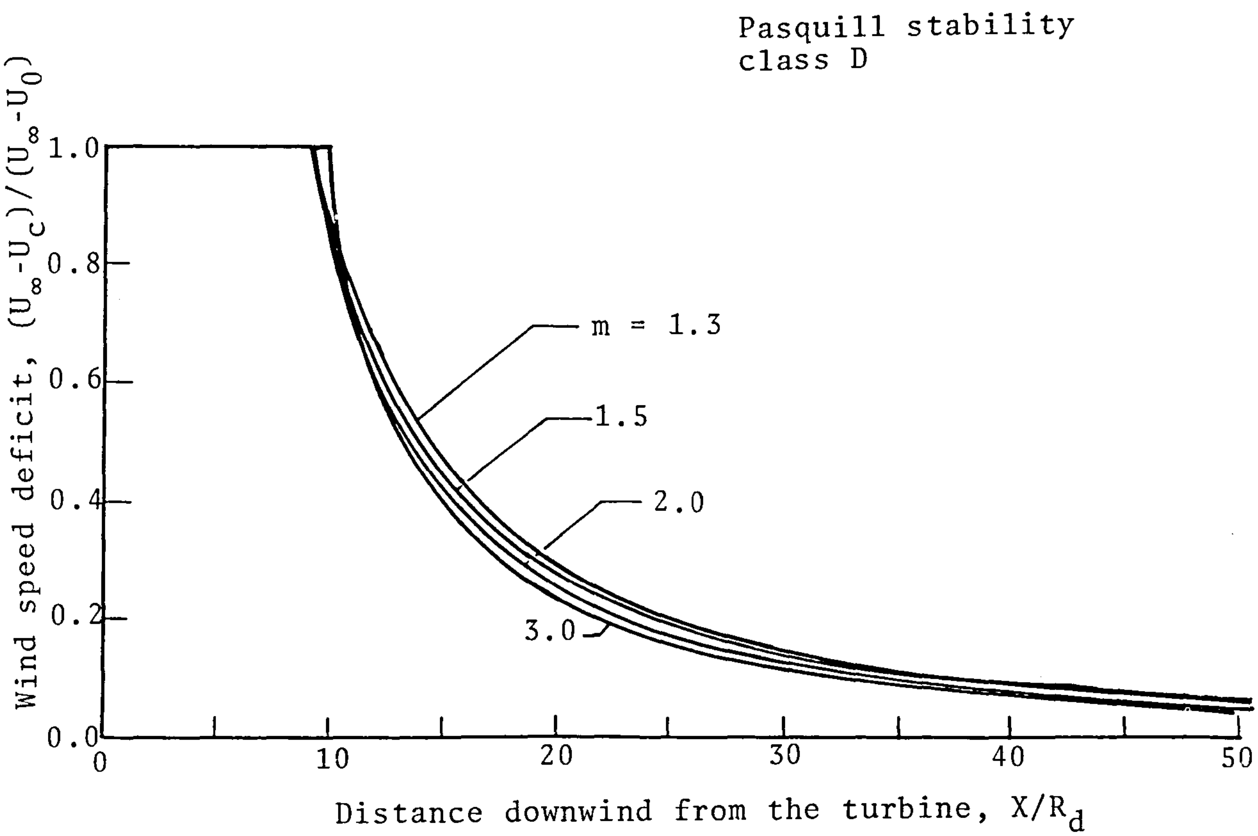

Figure 5-13. Effect of initial velocity ratio in the wake on wind speed deficit parameter.

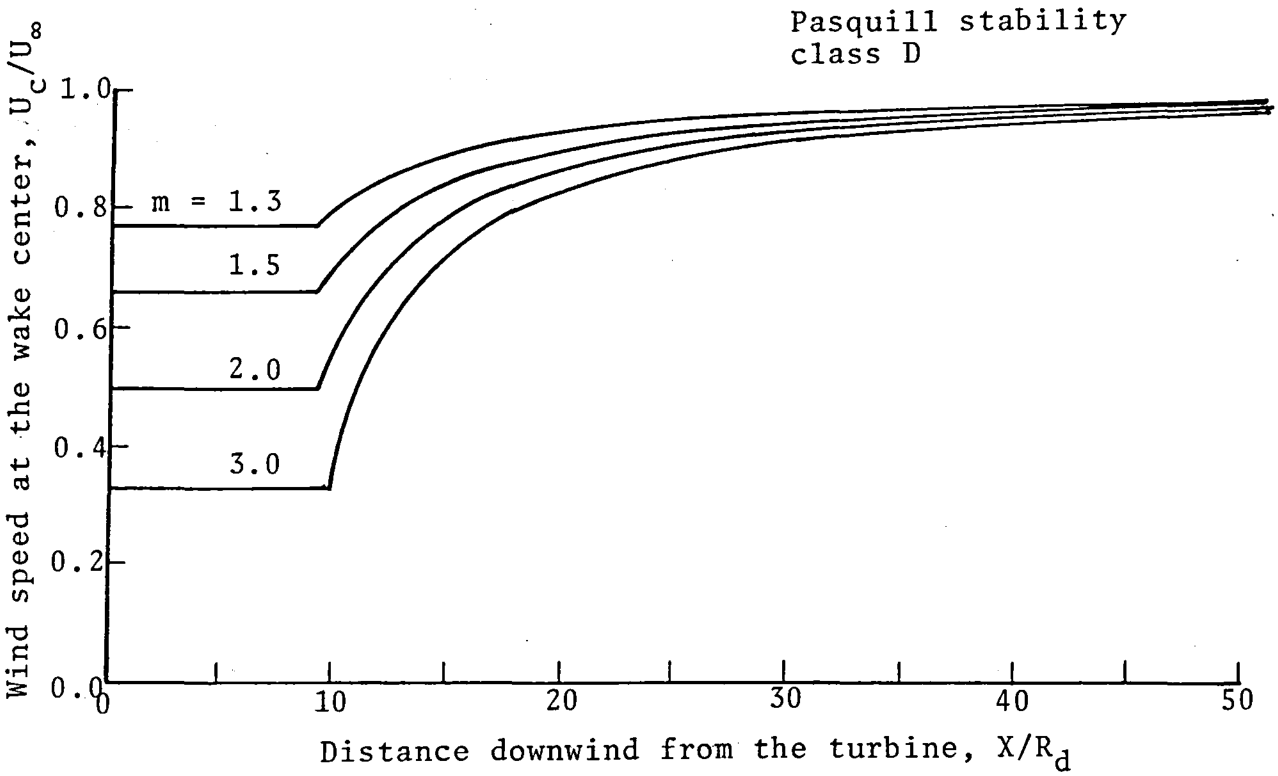

Figure 5-14. Effect of initial velocity ratio in the wake on wind speed at the wake center.

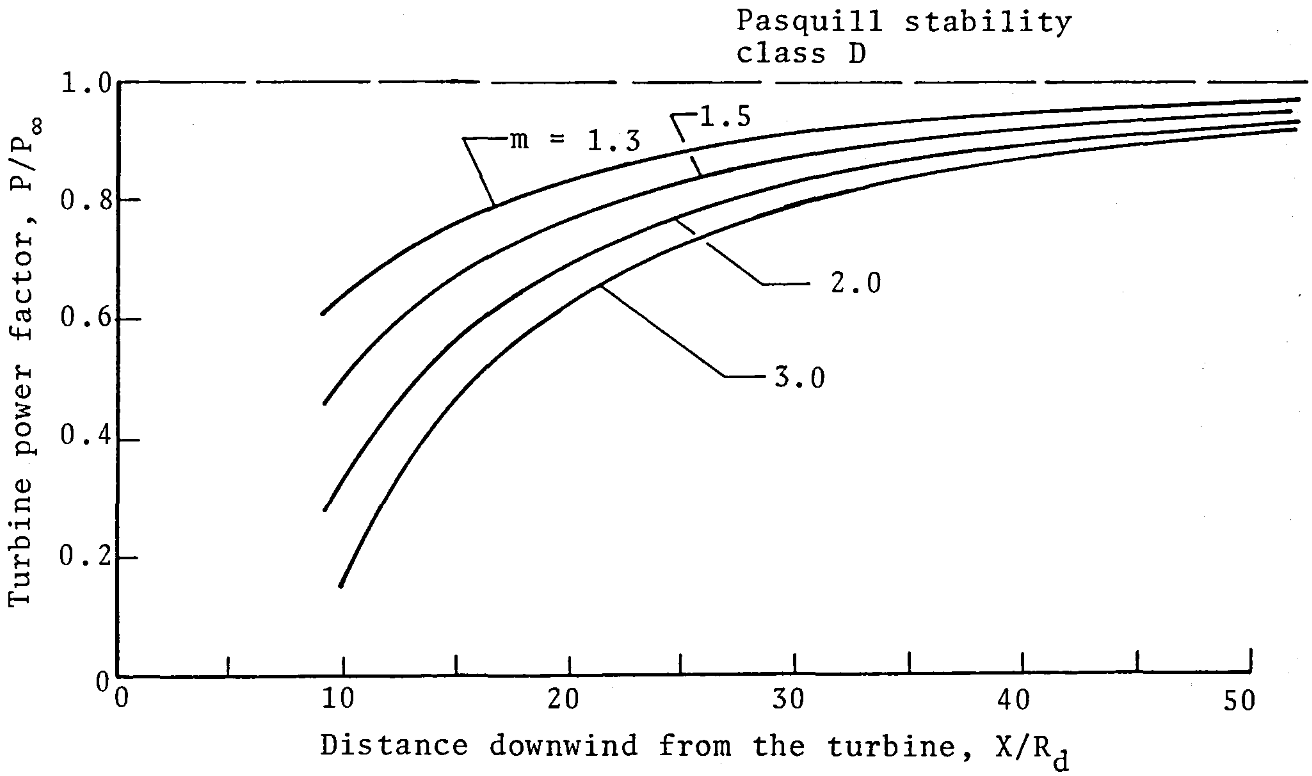

Figure 5-15. Effect of initial velocity ratio in the wake on turbine power factor.

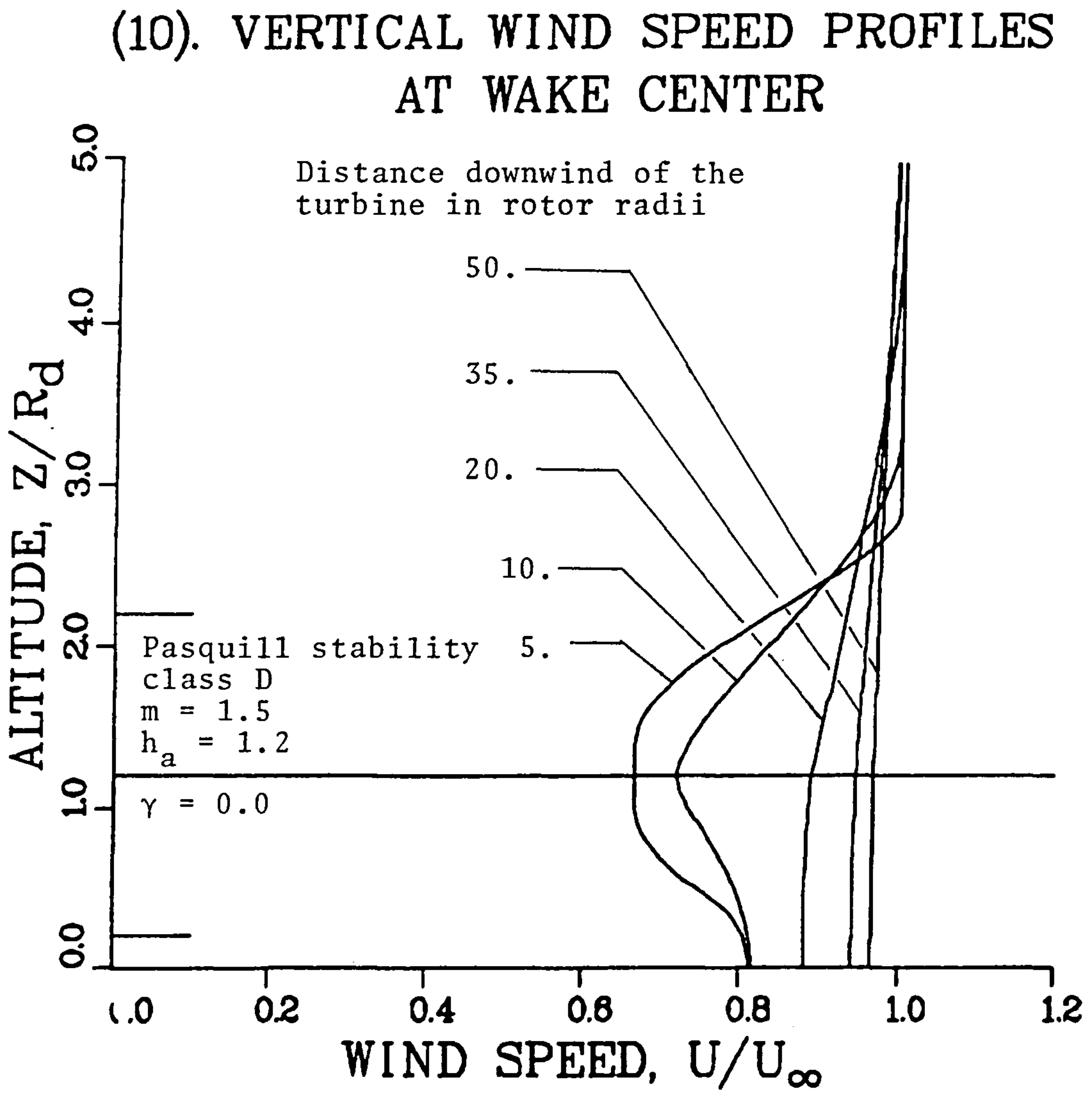

Figure 5-16. Effect of the gound plane on the vertical wind speed profile for a rotor hub altitude of 1.2 rotor radii.

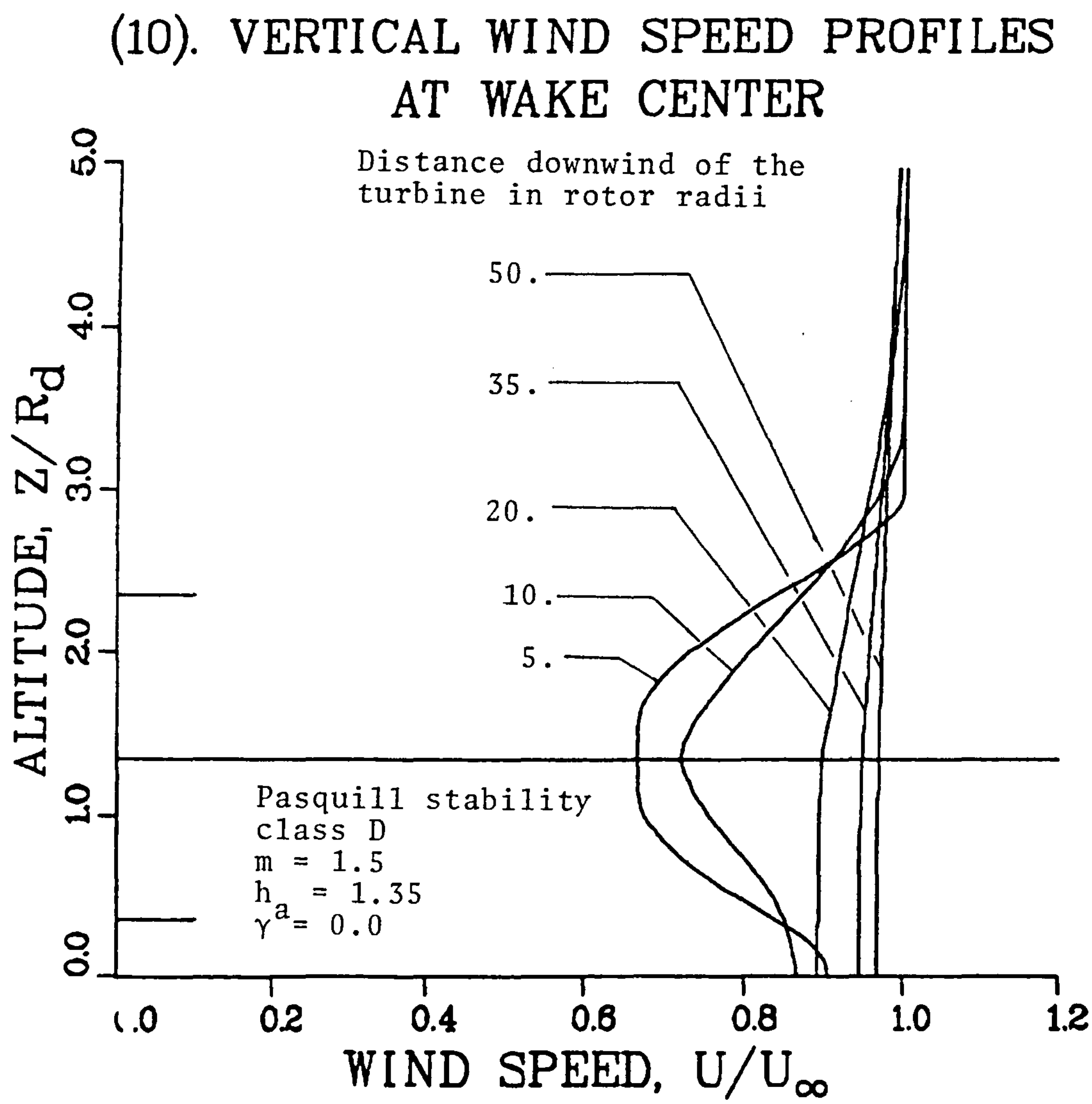

Figure 5-17. Effect of the ground plane on the vertical wind speed profile for a rotor hub altitude of 1.35 rotor radii.

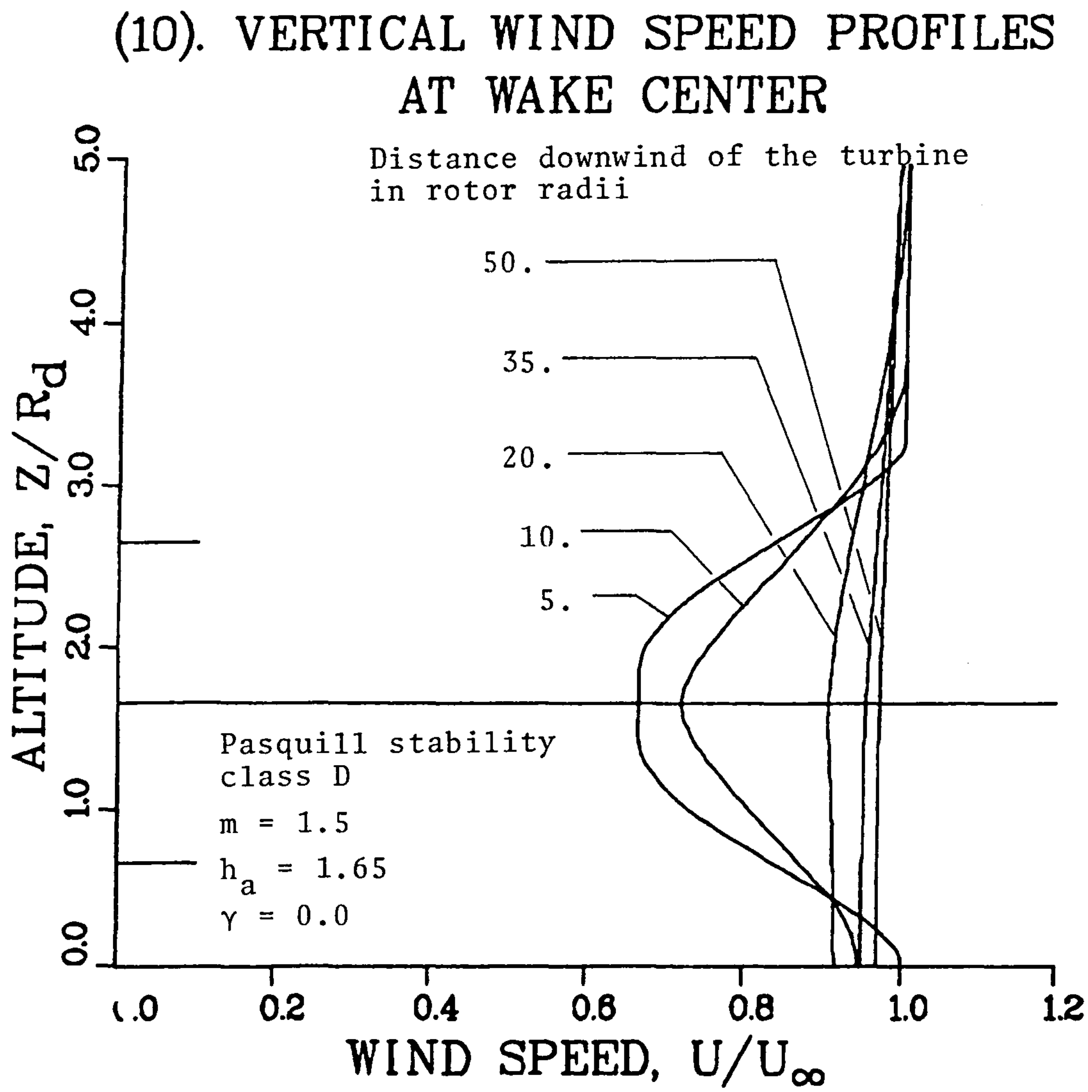

Figure 5-18. Effect of the ground plane on the vertical wind speed profile for a rotor hub altitude of 1.65 rotor radii.

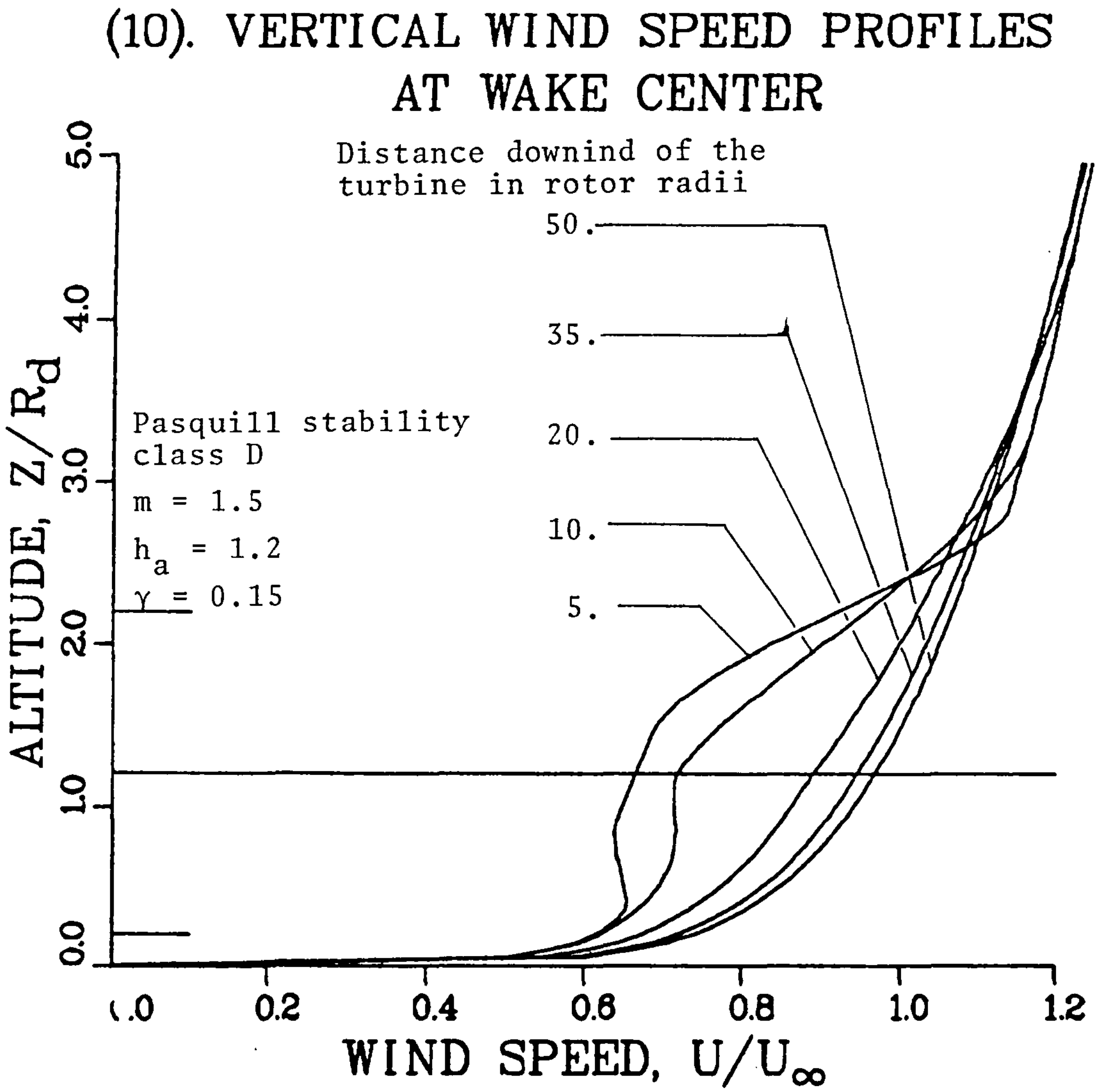

Figure 5-19. Effect of the ground plane on the vertical wind speed profile for a positive wind speed profile exponent and a rotor hub altitude of 1.2 rotor radii.

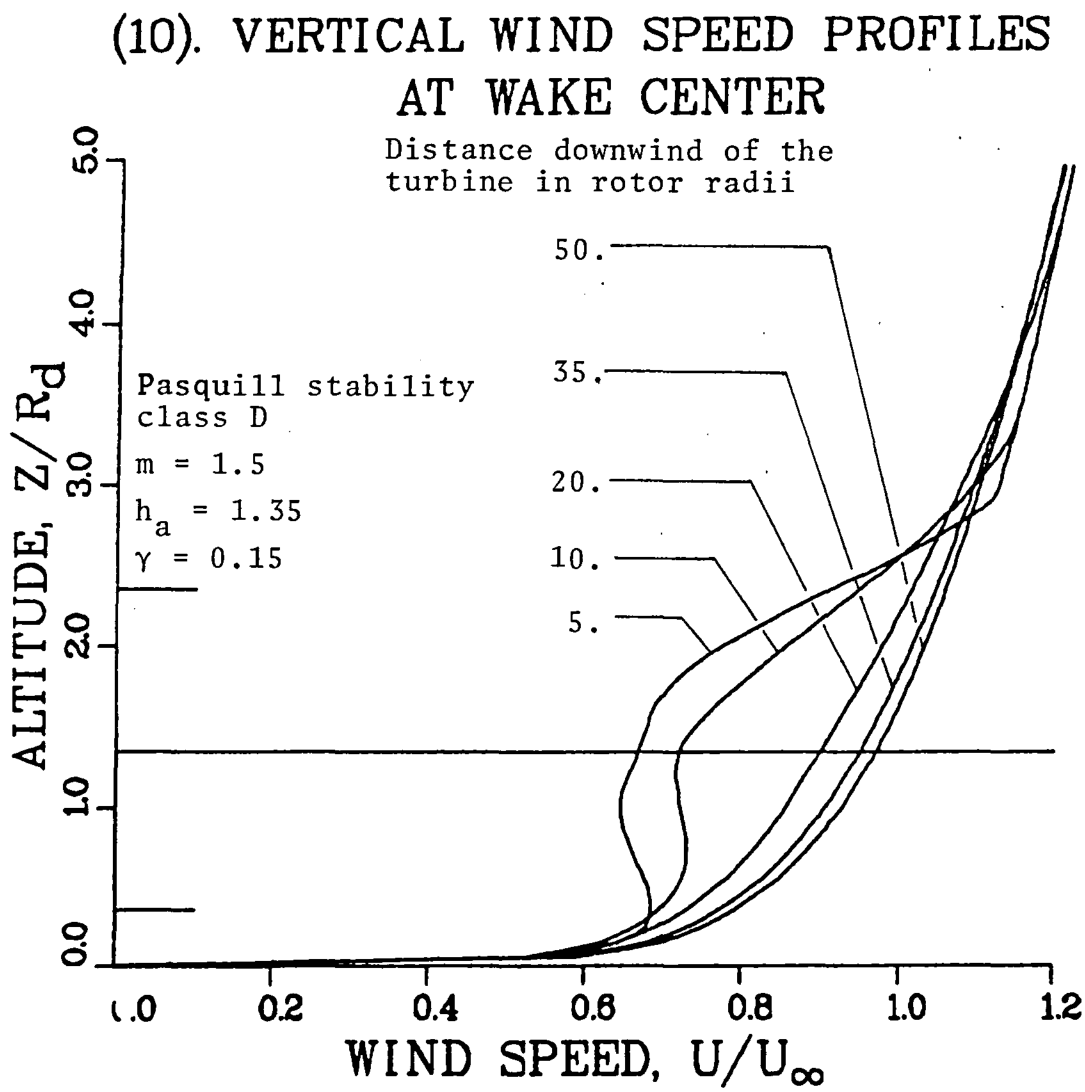

Figure 5-20. Effect of the ground plane on the vertical wind speed profile for a positive wind speed profile exponent and a rotor hub altitude of 1.35 rotor radii.

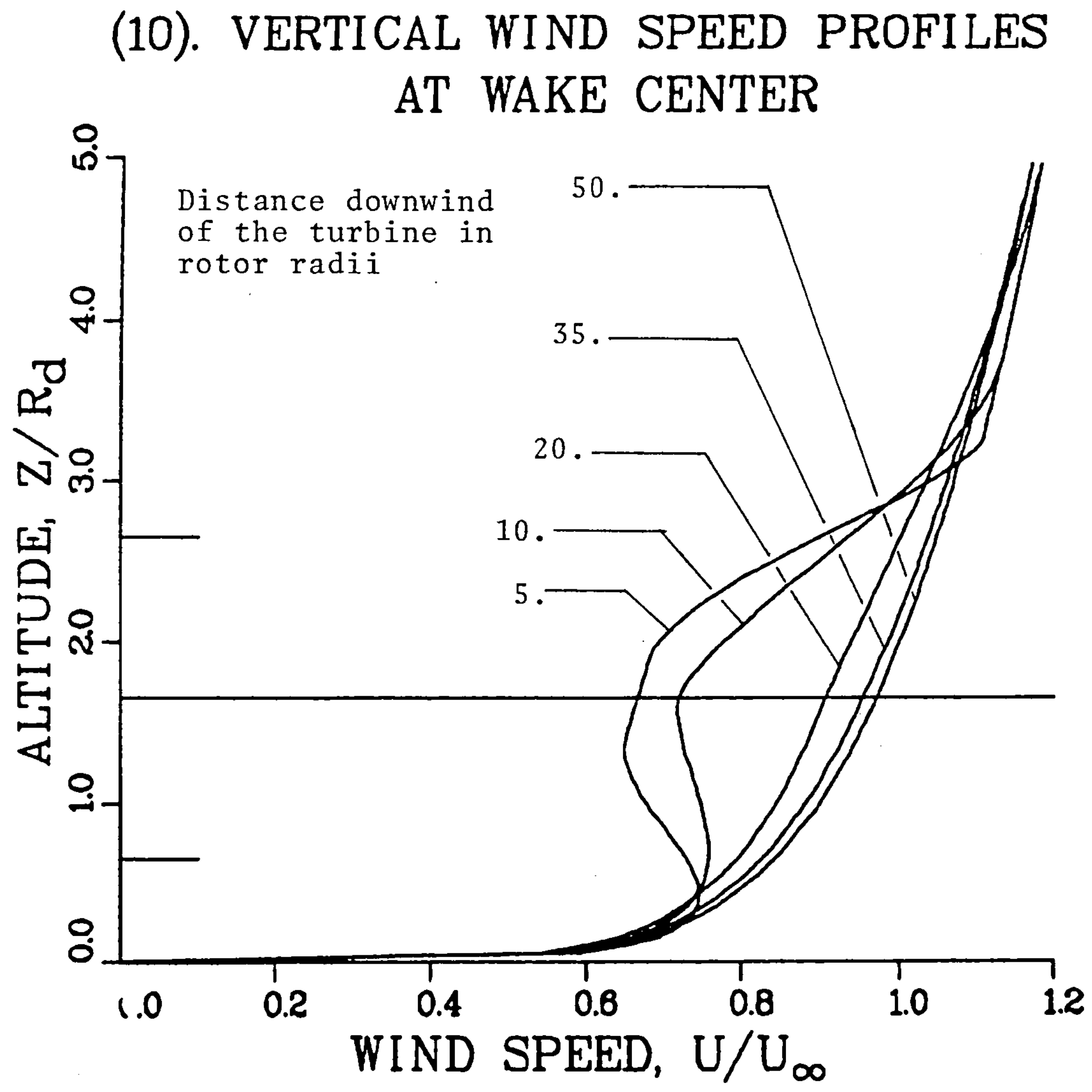

Figure 5-21. Effect of the ground plane on the vertical wind speed profile for a positive wind speed profile exponent and a rotor hub altitude of 1.65 rotor radii.

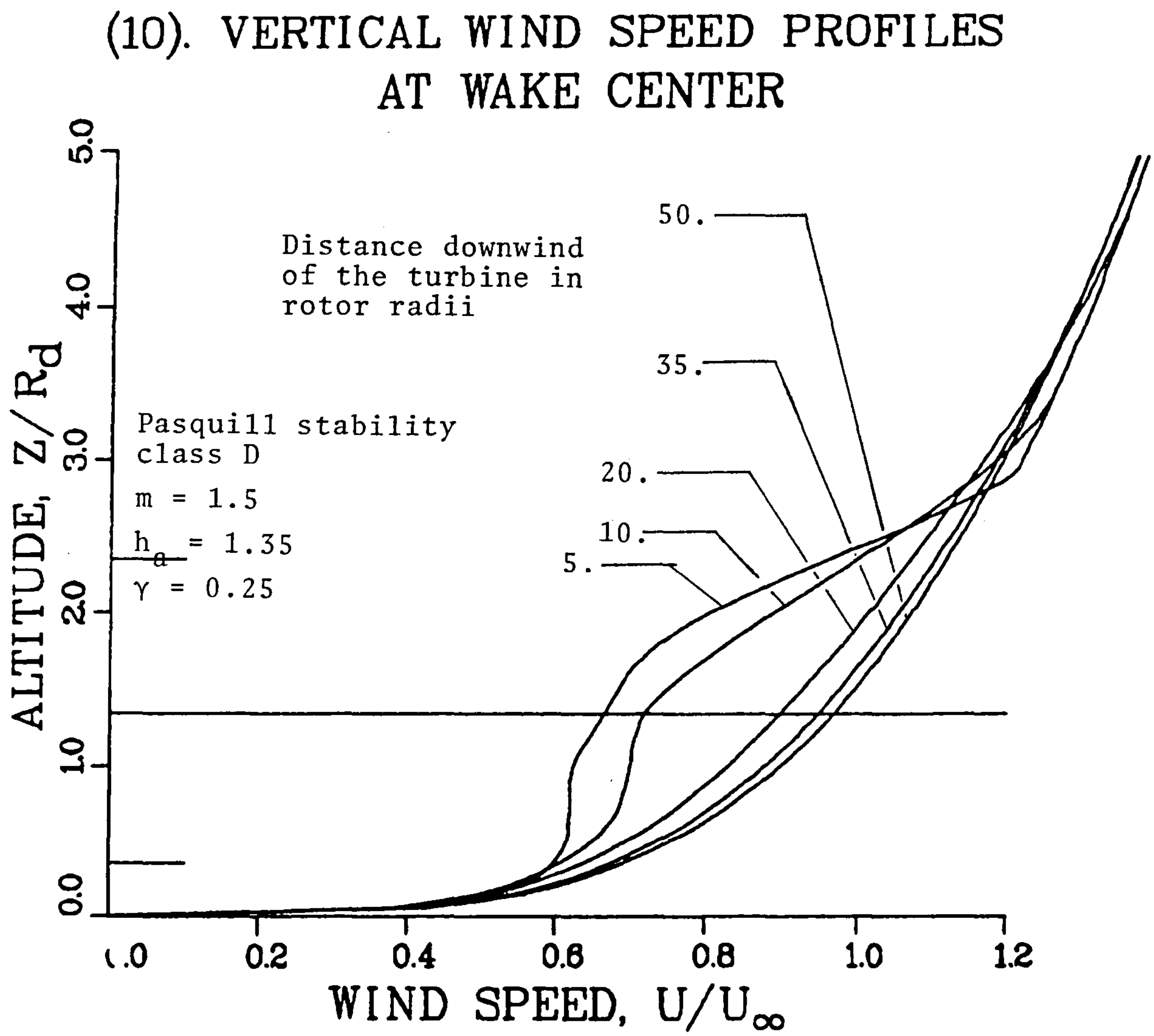

Figure 5-22. Effect of the ground plane on the vertical wind speed profile for a power law exponent for the free stream flow of 0.25.

6.0 CONCLUDING REMARKS

An analytic model for the calculation of the recovery of wakes of large wind turbines has been developed. The model is based upon the theory of coflowing turbulent jets as developed by G. N. Abramovich. The Abramovich model has been modified to relate turbine parameters to the wake parameters used by Abramovich, to add the effects of ambient turbulence to the model, and to calculate the turbine power factor, which is defined as the ratio of the power which a turbine in the wake of another turbine would generate the power which the turbine would generate if it were in the free stream air flow. The theory for the dispersion of pollutants in a turbulent atmosphere as developed by F. Pasquill has been adapted to describe wake growth due to ambient turbulence, and the methods used by P. B. S. Lissaman have been used to combine the effects of wake growth as described by the Abramovich model with wake growth due to ambient turbulence.

This approach is currently considered to be the most appropriate approach for an analytical model of turbine wake recovery. However, several intrinsic uncertainties about the model remain. A numberical approach (e.g., finite difference integration of the three-dimensional Navier-Stokes equations) could be used. However, it is not considered appropriate, because comparable gross uncertainties would exist for this approach (e.g., magnitude of the turbulent fluctuations) and large costs would be incurred to generate solutions. Any further work on the model might best be directed toward expansions or refinements of factors within the existing formulation.

The literature has shown only very limited experimental evaluation of the model. Comparisons of computer results with experimental data are therefore, very much desired to establish the adequacy of the model and to resolve the questions involved. However, it is recognized that extensive data from full scale turbines are very difficult to obtain. An attempt to obtain such data is described in the companion report on the second (experimental) phase of the research program conducted by Lockheed. Wind turbine model tests in wind tunnels may also be of use in validating some aspects of the model formulation. Although it would be almost impossible to duplicate the Reynolds number and geometry of an actual turbine in the wind tunnel, meaningful applicable results could be obtained. In another approach, the computer program may be used to determine the sensitivity of the final results to the factors in question.

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Appendix A

LIST OF SYMBOLS

This appendix contains a list of symbols used in the development of the analytic model. Where appropriate, variable names used in the FORTRAN computer program are also listed. The equation number, figure number, or table number following the definition is the equation, figure, or table where the parameter is defined or first used in the description of the analytic model.

A few parameters used in the deriviation have been omitted from this list. They are omitted if they are of minor importance and are used for only two or three successive equations in the derivation so that the location of their definition in the text is never in doubt.

Symbol	Computer Symbol	Definition
A		Cross-sectional area of the turbine disk. Equation (3-1).
A_∞		Free stream cross-sectional area of the stream tube which passses through the turbine disk. Equation (3-64).
a		Axial induction factor for the turbine. Equation (3-7).
B	B	Parameter used for the derivation of the turbine power factor. Equation (3-75).
b		Full width of the boundary layer in region I. Equation (3-36).
b_1, b_2		Partial widths of the boundary layer in region I. Figure 3-5.
c		Growth rate of the boundary layer in region I. Equation (3-59).
H_a	AH	Hub altitude of the turbine in physical units. Table 3-2.
h_a	AH	Hub altitude of the turbine normalized by the rotor radius. Table 3-2.

Symbol	Computer Symbol	Definition
h	AH	Height of the top of the wake above the ground. Equation (3-80).
I	IO	Input parameter which specifies whether geometric input parameters are in physical units or normalized by the rotor radius. Table 3-2.
i		Index to define altitude of lateral wind speed profile. Equation (5-1).
J	J	Number of downwind locations at which wind speed profiles are to be calculated. Table 3-2.
K		Constant used in derivation of the turbine power factor. Equation (3-64).
m	CPM	Ratio of the free stream wind speed to the initial wind speed in the wake (after expansion by potential effects to wake radius, R_0). Equation (3-9).
$\dot{m}$		Mass flow rate of air passing through the turbine disk. Equation (3-1).
n	AN	Ratio of the downwind extent of Region II to the downwind extent of Region I. Equation (3-47).
P		Power extracted from the air by the turbine. Equation (3-63).
P_∞		Power extracted from the air by a turbine in the free stream. Equation (3-10).
$\overline{P}$	PR	Turbine power factor, which is the ratio of power extracted by a turbine in the wake of another turbine to the power which an identical turbine would extract in the free stream flow. Equation (3-63).

Symbol	Computer Symbol	Definition
p^+		Static pressure on the upwind side of the turbine. Equation (3-2).
p^-		Static pressure on the downwind side of the turbine. Equation (3-2).
p_∞		Ambient static pressure. Equation (3-3).
R		Radial coordinate in physical units.
R_d	RRR	Radius of the turbine rotor disk. Figure 3-1.
R_0		Initial wake radius in physical units. Figure 3-1.
R_1		Radius of the potential core in Region I. Figure 3-1.
R_2		Wake radius in physical units. Figure 3-1.
R_{21}		Wake radius at the end of Region I in physical units. Figure 3-1.
R_{22}		Wake radius at the end of Region II in physical units. Figure 3-1.
R_∞		Free stream radius of the stream tube which passes through the turbine disk. Figure 3-6.
r		Radial coordinate normalized by the turbine disk radius.
r_{sj}	R2SAV	Wake radius at the jth downwind location at which wind speed profiles are to be plotted. Equation (3-84).
r_0	R0	Initial wake radius normalized by the turbine disk radius. Equation (3-18).

Symbol	Computer Symbol	Definition
r_1	R1	Radius of the potential core in Region I normalized by the turbine disk radius. Equation (3-36).
r_2	R2	Wake radius normalized by the turbine disk radius. Equation (3-24).
r_2'	R2P	Incremented value of R2 used in numerical inetgration for Region III. Discussion preceeding Equation (3-51).
r_{21}	R21	Wake radius at the end of Region I normalized by the turbine disk radius. Equation (3-34).
r_{22}	R22	Wake radius at the end of Region II normalized by the turbine disk radius. Equation (3-50).
r_∞	RI	Free stream radius (normalized by the turbine disk radius) of the stream tube which passes through the turbine disk. Equation (3-67).
Δr	DR	Increment of r used for numerical integration in Region III. Equation (3-62).
T		Axial thrust on the turbine disk. Equation (3-1).
U		Local wind speed in the wake. Equation (3-32).
U_c		Wind speed at the center of the wake. Equation (3-81).
U_T		Wind speed of the air passing through the turbine disk. Equation (3-1).
U_0		Initial wind speed of the wake (after expansion to wake radius, R_0). Equation (3-1).

Symbol	Computer Symbol	Definition
U_∞		Free stream wind speed. Equation (3-1).
ΔU_c		Wind speed deficit at the center of the wake in physical units. Equation (3-53).
Δu		Wind speed deficit in the wake normalized by the initial wind speed deficit. Equation (3-22).
u_c		Wind speed at the center of the wake normalized by the free stream wind speed. Equation (3-82).
Δu_c	DUC	Wind speed deficit at the center of the wake normalized by the initial wind speed deficit at the center of the wake. Equation (3-22).
u	UB	Wind speed in the wake normalized by the free stream wind speed. Equation (3-73).
u^*	UBG	Wind speed in the wake normalized by the free stream wind speed and calculated for the image turbine. Equation (3-96).
w		Wake width normalized by the rotor radius. Equation (3-79).
X		Coordinate distance measured downwind from the turbine in physical units.
X_{dj}	XNPT	Downwind location at which wind speed profiles are to be calculated. Table 3-2.

Symbol	Computer Symbol	Definition
X_H		Downwind extent of Region I in physical units. Figure 3-1.
X_N		Downwind extent of Region II in physical units. Figure 3-1.
x	X	Coordinate distance measured downwind from the turbine, normalized by the turbine disk radius. Equation (3-20).
x'	XP	Incremented value of x used during the numerical integration in Region III. Equation (3-62).
x_H	XH	Downwind extent of Region I normalized by the turbine disk radius. Equation (3-39).
$(x_H)_m$	XHM	Downwind extent of Region I from Abramovich solution normalized by the turbine disk radius. Equation (3-47).
x_{dj}	XNPT	Downwind location at which wind speed profiles are to be calculated, normalized by the rotor radius. Equation (3-83).
Y		Lateral coordinate measured from the vertical center of the wake in physical units.
ΔY	DYY	Increment in the lateral coordinate used in generating the wake profile. in physical units. Table 3-2.
Δy	DYY	Increment in the lateral coordinate used in generating the wake profile, normalized by the turbine rotor radius. Table 3-2.
Z		Vertical coordinate measured positive upward from the ground in physical units.

Symbol	Computer Symbol	Definition
z	Z	Vertical coordinated normalized by the turbine rotor radius.
z_v	ZVAL	Altitude relative to the turbine hub, normalized by the rotor radius. Equation (3-85).
Z_0	Z0	Altitude of the lowest wind speed profile in physical units. Table 3-2.
ΔZ	DZZ	Altitude increment between successive wind speed profiles in physical units. Table 3-2.
Δz	DZZ	Altitude increment between successive wind speed profiles normalized by the rotor radius. Table 3-2.
α	ALPHA	Wake growth rate due to ambient turbulence. Equation (3-28).
γ		Exponent for the power law wind speed profile. Table 3-2.
ΔKE		Change in kinetic energy of the air passing through the turbine. Equation (3-10).
η	N	Non-dimentional radial parameter used in Region I. Equation (3-33).
ρ		Mass density of ambient air. Equation 3-1.
σ		Pollution dispersion coefficient from plume theory. Equation (3-19).
σ_θ	ST	Standard deviation of wind direction in the ambient air. Table 3-2.

Appendix B

PROGRAM LISTING

The following pages contain a listing of the FORTRAN computer program used to calculate turbine wake progerties and to generate plots of wake properties. Comments in the program listing describe the sequence of calculations. Equation numbers given in the listing give the equation numbers in the text of this report.

The main program is listed first with suboutines following. Subroutine R3 calculates the wind speed deficit parameter, Δu_c; the wake growth rate due to mechanical turbulence, $(dr_2/dx)_m$; and the turbine power factor, $\overline{P}$, in Region III. Subroutine CALCU is used to calculate the wind speed profiles. The remaining suboutines are plotting suboutines used to generate appropriate plots. Subroutine XVSR2 is used to plot wake boundary parameters as a function of the downwind coordinate, x. Subroutine XVSDUC is used to plot the wind speed deficit parameter, Δu_c, as a function of x. Subroutine XVSUB is used to plot the wind speed at the wake center as a function of x. Subroutine XVSPR is used to plot the turbine power factor, $\overline{P}$, as a function of x. Subroutine YVSUBR is used to generate the plots for the lateral wind speed profiles. Subroutine UVSZ is used to generate the plots for the vertical wind speed profiles.

```
C
C
C          TITLE : WAKE
C
C        AUTHORS : DR. W. EBERLE
C                  A. C. SCOPPA
C                  LOCKHEED MISSILES AND SPACE CO.
C
C           DATE : WINTER, 1981
C
C . . . THIS PROGRAM CALCULATES WAKE PARAMETERS FOR THE RECOVERY
C . . . OF WAKES OF LARGE WIND TURBINES. THE PROGRAM PLOTS WAKE
C . . . RADIUS, WAKE WIDTH, WAKE ALTITUDE, THE WIND SPEED DEFICIT
C . . . AT THE CENTER OF THE WAKE, THE WIND SPEED AT THE CENTER
C . . . OF THE WAKE, AND THE TURBINE POWER FACTOR AS FUNCTIONS OF
C . . . THE DISTANCE DOWNWIND OF THE TURBINE. IN ADDITION, WIND SPEED
C . . . PROFILES OF THE WIND SPEED AS A FUNCTION OF THE LATERAL
C . . . COORDINATE ARE PLOTTED FOR EIGHT ALTITUDES FOR EACH DOWNWIND
C . . . LOCATION SELECTED BY THE USER. A VERTICAL WIND SPEED PROFILE
C . . . IS GENERATED FOR EACH DOWNWIND LOCATION SELECTED FOR PLOTS OF
C . . . OF LATERAL WIND SPEED PROFILES.
C
C
C
C
C . . . ST     - ATMOSPHERIC TURBULENCE PARAMATER.  IF INPUT AS A
C                POSITIVE NUMBER, IT IS THE STANDARD DEVIATION OF THE
C                WIND DIRECTION.  IF INPUT AS A NEGATIVE NUMBER, IT IS
C                THE PASQUILL STABILITY CLASS AS FOLLOWS:
C                -1 FOR PASQUILL STABILITY CLASS A
C                -2 FOR PASQUILL STABILITY CLASS B
C                -3 FOR PASQUILL STABILITY CLASS C
C                -4 FOR PASQUILL STABILITY CLASS D
C                -5 FOR PASQUILL STABILITY CLASS E
C                -6 FOR PASQUILL STABILITY CLASS F
C                IF ST IS INPUT AS 0., AN AMBIENT TURBULENCE OF ZERO
C                IS USED, AND THE RESULTING PROFILES ARE THE
C                ABRAMOVICH SOLUTIONS.
C
C . . . CPM    - RATIO OF FREE STREAM WIND SPEED TO INITIAL WIND SPEED
C                IN THE WAKE
C
C . . . AH     - HUB HEIGHT IN ROTOR RADII OR IN PHYSICAL UNITS AS
C                SPECIFIED BY IO
C
C . . . VH0    - AMBIENT WIND SPEED AT HUB ALTITUDE . THIS MAY BE GIVEN
C                IN PHYSICAL UNITS IF OUTPUT IS DESIRED IN PHYSICAL
C                UNITS . AN INPUT VALUE OF 1.0 WILL GIVE OUTPUT
C                NORMALIZED BY THE FREE STREAM WIND STREAM.
C
C . . . WEXP   - COEFFICIENT OF THE WIND SPEED POWER LAW OF THE FORM
C                  'V=V0*(Z/Z0)**WEXP'
C
C . . . RRR    - ROTOR RADIUS. THIS MAY BE GIVEN IN PHYSICAL UNITS.
C                AN INPUT VALUE OF 1.0 WILL GIVE ALL OUTPUT
```

```
C                 OF UNITS OF LENGTH IN ROTOR RADII. AN INPUT
C                 VALUE OF .5 WILL GIVE ALL OUTPUT OF UNITS
C                 OF LENGTH IN ROTOR DIAMETERS.  AN INPUT VALUE GREATER
C                 THAN 2.0 WILL GIVE ALL OUTPUT OF UNITS OF LENGTH IN THE
C                 PHYSICAL UNITS USED FOR THE ROTOR RADIUS.
C
C . . . J       - NUMBER OF DOWN WIND LOCATIONS AT WHICH WIND
C                 SPEED PROFILES ARE TO BE CALCULATED
C
C . . . IO      - INPUT OPTION FOR PARAMETERS WITH PHYSICAL DIMENSIONS
C                 (AH,XNPT,Z0,DZZ,DYY)
C                 1 FOR INPUT IN ROTOR RADII
C                 2 FOR INPUT IN PHYSICAL UNITS
C
C . . . NP      - SPECIFIES HOW VELOCITY IS TO BE NORMALIZED FOR
C                 PLOTS OF WIND SPEED PROFILES IN LATERAL DIRECTION.
C                 NP = 0 NORMALIZES WIND SPEED BY THE FREE STREAM WIND
C                 SPEED AT THAT ALTITUDE. NP = 1 NORMALIZES WIND SPEED
C                 BY THE FREE STREAM WIND SPEED AT THE HUB ALTITUDE.
C
C . . . XNPT    - DOWNWIND LOCATIONS AT WHICH WIND SPEED PROFILES
C                 ARE TO BE CALCULATED (ROTOR RADII OR PHYSICAL UNITS
C                 AS SPECIFIED BY IO
C
C . . . Z0      - MINIMUM ALTITUDE AT WHICH WIND SPEED PROFILES ARE TO BE
C               - CALCULATED (ROTOR RADII OR PHYSICAL UNITS AS SPECIFIED
C                 BY IO)
C
C . . . DZZ     - INCREMENT IN ALTITUDES AT WHICH WIND SPEED PROFILES ARE
C                 TO BE CALCULATED (ROTOR RADII OR PHYSICAL UNITS AS
C                 SPECIFIED BY IO)
C
C . . . DYY     - INCREMENT IN Y BY WHICH CALCULATIONS ARE TO BE MADE IN
C                 GENERATING THE WIND SPEED PROFILES (ROTOR RADII OR PHYSICAL
C                 UNITS AS SPECIFIED BY IO)
C
C        CONTROL INPUT :
C            RECORD 1 - (50F10.4)
C            [6 INPUT VARIABLES]
C            ST,CPM,AH,VH0,WEXP,RRR
C
C            RECORD 2 - (50I2)
C            [3 VARIABLES]
C            J,IO,NP
C
C            RECORD 3 - (50F10.4)
C            [J VARIABLES]
C            XNPT
C
C            RECORD 4 - (50F10.4)
C            [3 VARIABLES]
C            Z0,DZZ,DYY
C
C        OUTPUT FILES :
C            ONE DATA SET PER PLOT, NAMING CONVENTION 'SUBNAME.DAT'
```

```
C
C         MISCELLANEOUS :
C             THE PLOTTING PACKAGE 'DISSPLA' MUST BE LINKED WITH
C             'WAKE' BEFORE EXECUTION. THIS PROGRAM
C             CAN BE RUN 'AS IS' USING A PDP-10 COMPILER AND A
C             TEKTRONIX 4014 EXTENDED GRAPHICS TERMINAL, OTHERWISE
C             EXTENSIVE MODIFICATIONS WILL BE NECESSARY.
C
        COMMON /SUBCOM/ CPM,AH,R0,R2,DUC,UB,UBG,DRDX,PR,
     *                  XN,R22,R21,XH
C
        COMMON /PSCALE/ RRR,VH0
C
        COMMON /CRPA/ICRT,IPAPER
C
        DOUBLE PRECISION INFILE,UFILE
C
        REAL XNPT(20),R2SAV(20)
C
        DATA ICRT/1/,FLAG/-99./,E/2.7182818/
C
        DATA IUBAR/'UBAR'/
C
C . . . DETERMINE DATA SET SPECS FOR CONTROL INPUT
C
        WRITE(5,5)
5       FORMAT(' ENTER LOGICAL UNIT AND FILE NAME WHERE',
     *         ' CONTROL INPUT RESIDES :')
C
        READ(5,7)INU,INFILE
7       FORMAT(I2,A8)
C
C . . . OPEN DATA SET CONTAINING CONTROL INPUT
C
        OPEN (UNIT=INU,FILE=INFILE,ACCESS='SEQIN',DISPOSE='SAVE',
     *        DEVICE='DSK')
C
C . . . READ IN CONTROL DATA
C
        READ(INU,10) ST,CPM,AH,VH0,WEXP,RRR
10      FORMAT(50F10.4)
C
        READ(INU,20) J,IO,NP
20      FORMAT(50I2)
C
        READ(INU,10) (XNPT(I),I=1,J)
C
        READ(INU,10) Z0,DZZ,DYY
C
C . . . CALCULATE PARAMETERS IN ROTOR RADII IF THEY HAVE
C . . . BEEN INPUT IN PHYSICAL UNITS
C
        IF(.NOT.(IO .EQ. 0)) GO TO 25
C                                                  EQUATION (3-12)
            AH=AH/RRR
```

```
C                                                 EQUATION (3-13)
            Z0=Z0/RRR
C                                                 EQUATION (3-14)
            DZZ=DZZ/RRR
C                                                 EQUATION (3-15)
            DYY=DYY/RRR
            DO 26 I=1,J
C                                                 EQUATION (3-83)
               XNPT(I)=XNPT(I)/RRR
26          CONTINUE
25       CONTINUE
C
C . . .
C . . . CALCULATE CONSTANTS
C . . .
C
C . . .SET VALUE FOR ALPHA
C
30       CONTINUE
C                                                 EQUATION (3-28)
         IF(ST .GT. 0.)
     *      ALPHA=1.97*0.031*E**(.08*ST)
         IF(ABS(ST) .LT. 1.E-10)
     *      ALPHA=0.
         IF(ABS(ST+1.) .LT. 1.E-10)
     *      ALPHA=1.97*.212
         IF(ABS(ST+2.) .LT. 1.E-10)
     *      ALPHA=1.97*.156
         IF(ABS(ST+3.) .LT. 1.E-10)
     *      ALPHA=1.97*.104
         IF(ABS(ST+4.) .LT. 1.E-10)
     *      ALPHA=1.97*.069
         IF(ABS(ST+5.) .LT. 1.E-10)
     *      ALPHA=1.97*.050
         IF(ABS(ST+6.) .LT. 1.E-10)
     *      ALPHA=1.97*.034
35       CONTINUE
C
C . . . SET VALUE FOR RADIUS CHANGE
C
         DR=.02
C
C . . . CALCULATE INITIAL WAKE RADIUS IN ROTOR RADII
C
C                                                 EQUATION (3-18)
         R0=SQRT((CPM+1.)/2.)
C
C . . . CALCULATE WAKE RADIUS AT END OF REGION I
C
C                                                 EQUATION (3-34)
         R21=R0/SQRT(.214+.144*CPM)
C
C . . . CALCULATE DOWNWIND EXTENT OF REGION I BY ABRAMOVICH
C
C                                                 EQUATION (3-35)
```

```
      XHM=R0*(1.+CPM)/
     *     (.27*(CPM-1)*SQRT(.214+.144*CPM))
C
C . . . CALCULATE DOWNWIND EXTENT OF REGION I IN PRESENCE OF
C . . . AMBIENT TURBULENCE
C
           XH=0.5*R21/SQRT((0.5*R21/XHM)**2+ALPHA**2)
C
C . . . CALCULATE WAKE PARAMETER AT THE END OF REGION II
C
C                                               EQUATION (3-47)
      AN=SQRT(.214+.144*CPM)*
     *    (1.-SQRT(.134+.124*CPM))/
     *    ((1.-SQRT(.214+.144*CPM))*
     *    SQRT(.134+.124*CPM))
C                                               EQUATION (3-46)
      XN=AN*XH
C                                               EQUATION (3-49)
      R22=R0+AN*(R21-R0)
      X=XN
      R2=R22
      CALL R3
40    CONTINUE
C
C . . . SAVE VALUES OF WAKE RADIUS AT INPUT VALUES OF
C . . . XNPT(I) FOR THE XNPT(I) WHICH LIES IN REGIONS I OR III
C
      DO 55 I=1,J
          IF(.NOT.(XNPT(I) .LE. XN)) GO TO 54
C                                               EQUATION (3-84)
              R2SAV(I)=R0+(R22-R0)*XNPT(I)/XN
54        CONTINUE
55    CONTINUE
C
C . . .
C . . . CREATE FIRST SET OF PLOT FILES FOR WAKE RADIUS,
C . . . WAKE WIDTH, WAKE ALTITUDE, WIND SPEED DEFICIT AT THE CENTER
C . . . OF THE WAKE, WIND SPEED AT THE CENTER OF THE WAKE, AND
C . . . THE TURBINE POWER FACTOR AS A FUNCTION OF THE DISTANCE
C . . . DOWN WIND OF THE TURBINE
C . . .
C
      OPEN(UNIT=10,DEVICE='DSK',ACCESS='SEQOUT',
     *     MODE='ASCII',DISPOSE='SAVE',FILE='XVSR2.DAT')
      OPEN(UNIT=11,DEVICE='DSK',ACCESS='SEQOUT',
     *     MODE='ASCII',DISPOSE='SAVE',FILE='XVSDUC.DAT')
      OPEN(UNIT=12,DEVICE='DSK',ACCESS='SEQOUT',
     *     MODE='ASCII',DISPOSE='SAVE',FILE='XVSUB.DAT')
      OPEN(UNIT=13,DEVICE='DSK',ACCESS='SEQOUT',
     *     MODE='ASCII',DISPOSE='SAVE',FILE='XVSUBG.DAT')
      OPEN(UNIT=14,DEVICE='DSK',ACCESS='SEQOUT',
     *     MODE='ASCII',DISPOSE='SAVE',FILE='XVSPR.DAT')
C
C
      FIRSTX=0.
```

```
      FIRSTY=R0
      FY2=R0+AH
      FY3=R0*2
C
      WRITE(10,10) FIRSTX,FIRSTY,FY2,FY3
C
      FIRSTY=1.
C
      WRITE(11,10) FIRSTX,FIRSTY
C
      FIRSTY=1./CPM
C
      WRITE(12,10) FIRSTX,FIRSTY
C
      FIRSTY=1./CPM
C
      WRITE(13,10) FIRSTX,FIRSTY
C
      XOLD=X
      R2OLD=R2
C
C . . . SEARCH THROUGH X, TERMINATE WHEN R2 IS GREATER THAN 8 ,
C . . . OR X IS GREATER THAN 52

C
60    CONTINUE
      IF(.NOT.(R2 .LE. 8.  .AND.  X .LE. 52.)) GO TO 70
          IF(.NOT.(X .EQ. 0.))GO TO 61
              R2=R0
              DUC=1.
              UB=1./CPM
              UBG=1./CPM
61        CONTINUE
          IF(.NOT.(X .EQ. XN)) GO TO 62
              R2=R22
              DUC=1.
              UB=1./CPM
              UBG=1./CPM
62        CONTINUE
          RADDAH=R2+AH
          RMUL2=R2*2.
          WRITE(10,10) X,R2,RADDAH,RMUL2
          WRITE(11,10) X,DUC
          WRITE(12,10) X,UB
          WRITE(13,10) X,UBG
          IF(.NOT.(X .NE. 0.)) GO TO 63
              WRITE(14,10) X,PR
63        CONTINUE
          DO 69 I=1,J
              IF(.NOT.(XNPT(I) .GT. XN)) GO TO 68
                  IF(.NOT.(X .GT. XNPT(I)
     *                      .AND.
     *               XOLD .LT. XNPT(I))) GO TO 66
                      R2NEW=((R2-R2OLD)/(X-XOLD))*(XNPT(I)-XOLD)+R2OLD
                      R2SAV(I)=R2NEW
```

```
66                  CONTINUE
                    IF(.NOT.(X .EQ. XNPT(I))) GO TO 67
                        R2SAV(I)=R2
67                  CONTINUE
68              CONTINUE
69          CONTINUE
            XOLD=X
            R2OLD=R2
            R2P=R2+DR
            CALL R3
            DRDXE=SQRT(((DRDX+DRDXP)/2.)**2+ALPHA**2)
            XP=X+DR/DRDXE
            R2=R2P
            X=XP
            DRDX=DRDXP
            GO TO 60
70      CONTINUE
C
C . . . CLOSE FILES
C
        CLOSE(UNIT=10)
        CLOSE(UNIT=11)
        CLOSE(UNIT=12)
        CLOSE(UNIT=13)
        CLOSE(UNIT=14)
C
C . . . INITIALIZE PLOTTER FOR DISSPLA, 1200 BAUD RATE, EXTENDED
C . . . GRAPHICS TERMINAL
C
        CALL TKTRN(120,1)
C
C . . . PRODUCE PLOTS
C
        CALL XVSR2
        CALL XVSDUC
        CALL XVSUB
C        CALL XVSUBG
        CALL XVSPR
C
C . . .
C . . . GENERATE WIND SPEED PROFILE PLOTS AS A FUNCTION OF THE
C . . . LATERAL COORDINATE, FOR EIGHT ALTITUDES FOR EACH
C . . . DOWNWIND LOCATION SELECTED BY THE USER. ALSO, A
C . . . VERTICAL WIND SPEED PROFILE IS GENERATED FOR EACH
C . . . DOWNWIND LOCATION SELECTED.
C . . .
C
        IUNITZ=J+14+1
        OPEN(UNIT=IUNITZ,FILE='UVSZ',DEVICE='DSK',MODE='ASCII',
     *       DISPOSE='SAVE',ACCESS='SEQOUT')
        DO 160 I=1,J
            WRITE(IUNITZ,10) FLAG
            IUNIT=I+14
            ENCODE(6,75,UFILE) IUBAR,IUNIT
75          FORMAT(A4,I2)
```

```
            OPEN(UNIT=IUNIT,DEVICE='DSK',ACCESS='SEQOUT',
     *           MODE='ASCII',DISPOSE='SAVE',FILE=UFILE)
            Z=Z0
            Y=0.
C
C . . . STEP THROUGH EIGHT ALTITUDES
C
            DO 140 JJ=1,8
                WRITE(IUNIT,10) FLAG
                Y=0.
                UBAR=0.
 90             CONTINUE
                IF(.NOT.(ABS(1.-UBAR) .GT. .00001)) GO TO 130
                    YVAL=Y
C                                                  EQUATION (3-85)
                    ZVAL=AH-Z
                    CALL CALCU(YVAL,ZVAL,XNPT,R2SAV,I,UBARA)
                    YVAL=Y
C                                                  EQUATION (3-95)
                    ZVAL=AH+Z
                    CALL CALCU(YVAL,ZVAL,XNPT,R2SAV,I,UBARB)
C                                                  EQUATION (3-98)
                    IF(NP .EQ. 0)
     *                  UBAR=UH0*(UBARA+UBARB-1.)
C                                        EQUATIONS (3-98) AND (3-99)
                    IF(NP .EQ. 1)
     *                  UBAR=UH0*(UBARA+UBARB-1.)*
     *                        (Z/AH)**UEXP
                    WRITE(IUNIT,10) Y,UBAR
                    Y=Y+DYY
                    GO TO 90
130             CONTINUE
                Y=6.
                UBAR=1.
                WRITE(IUNIT,10)Y,UBAR
                Z=Z+DZZ
140         CONTINUE
C
C . . . CALCULATE VERTICAL WIND SPEED PROFILES
C
            YVAL=0.
            Z=0.
145         CONTINUE
            IF(.NOT.(Z .LE. 5.))GO TO 150
C                                                  EQUATION (3-85)
                ZVAL=AH-Z
                CALL CALCU(YVAL,ZVAL,XNPT,R2SAV,I,UBARA)
                ZVAL=AH+Z
C                                                  EQUATION (3-95)
                CALL CALCU(YVAL,ZVAL,XNPT,R2SAV,I,UBARB)
C                                        EQUATIONS (3-98) AND (3-99)
                UBAR=UH0*(UBARA+UBARB-1.)*(Z/AH)**UEXP
                WRITE(IUNITZ,10) UBAR,Z
                Z=Z+DYY
                GO TO 145
```

```
150         CONTINUE
C
            CLOSE(UNIT=IUNIT)
C
C . . . FOR EACH OF THE 'XNPT', PLOT LATERAL WIND SPEED PROFILES
C
            CALL YVSUBR(IUNIT,UFILE,XNPT,I)
C
160     CONTINUE
C
        CLOSE(UNIT=IUNITZ)
C
C . . . PLOT VERTICAL WIND SPEED PROFILES
C
        CALL UVSZ(IUNITZ,'UVSZ',AH,J)
C
C . . . TERMINATE PLOTTING SESSION, END PROGRAM
C
        CALL DONEPL
C
        END
```

```
      SUBROUTINE R3
C
C . . . THIS SUBROUTINE CALCULATES WAKE GROWTH RATE, WIND SPEED AT THE CENTER OF
C . . . THE WAKE, AND POWER RATIO IN REGION III OF THE WAKE.
C
      COMMON /SUBCOM/ CPM,AH,R0,R2,DUC,UB,UBG,DRDX,PR,
     *                XN,R22,R21,XH
C
C
C . . . CALCULATE WIND SPEED DEFICIT AT THE CENTER OF THE WAKE
C
C                                                       EQUATION (3-53)
      DUC=3.73*(-.258*CPM/(1.-CPM)-SQRT((.258*CPM/(1.-CPM))**2
     *    +.536/(R2*R2*(1.-CPM)/R0**2)))
C
C . . . CALCULATE WAKE GROWTH RATE DUE TO MECHANICAL TURBULENCE
C
C                                                       EQUATION (3-60)
      DRDX=.27/(2.*CPM/((CPM-1.)*DUC)-1.)
C
C . . . CALCULATE NON-DIMENSIONAL WIND SPEED AT THE WAKE CENTER
C                                                       EQUATION (3-94)
      UB=1.-DUC*(1.-1./CPM)
C
      UBG=UB
C
C . . . IS THE CENTER OF THE WAKE IN GROUND EFFECT ?
C
      IF(.NOT.(R2 .GT. 2.*AH)) GO TO 10
         UF=1.-DUC*(1.-1./CPM)*(1.-(2.*AH/R2)**1.5)**2
         UBG=UB*UF
10    CONTINUE
C
C . . . CALCULATE THE RATIO OF POWER GENERATED IN A GEOMETRICALLY
C . . . IDENTICAL TURBINE CENTERED IN THE WAKE OF THE UPWIND TURBINE
C . . . TO THE POWER THAT TURBINE WOULD GENERATE IF IT WERE IN THE
C . . . STREAM WIND
C                                                       EQUATION (3-75)
      B=DUC*(1.-1./CPM)
C                                                       EQUATION (3-78)
      F1=(1.-3.*B+3.*B**2-1.*B**3)/2.
      F2=(6.*B-12.*B**2+6.*B**3)/3.5
      F3=(-3.*B+18.*B**2-15.*B**3)/5.
      F4=(-12.*B**2+20.*B**3)/6.5
      F5=(3*B**2-15.*B**3)/8.
      F6=6.*B**3/9.5
      F7=B**3/(-11.)
      RI=SQRT((CPM+1.)/(2.*CPM))
      RIR=RI/R2
      PR=2.*(F1+F2*RIR**1.5+F3*RIR**3
     *   +F4*RIR**4.5+F5*RIR**6+F6*RIR**7.5+F7*RIR**9)
      RETURN
      END
```

```
      SUBROUTINE CALCU(Y,Z,XNPTDM,R2SVDM,I,UBAR)
C
C . . . THIS SUBROUTINE CALCULATES WIND SPEED IN THE WAKE FOR
C . . . GIVEN VALUES OF THE LATERAL AND VERTICAL COORDINATE
C . . . DISTANCES . THE FORM OF THE WIND SPEED PROFILE OF
C . . . REGION I , REGION II , OR REGION III AS APPROPRIATE
C
      COMMON /SUBCOM/ CPM,AH,R0,R2,DUC,UB,UBG,DRDX,PR,
     *                XN,R22,R21,XH
C
      REAL XNPTDM(20),N,R2SVDM(20)
C
      R=SQRT(Y**2+Z**2)                                  ! EQUATION (3-86)
C
C . . . DETERMINE IF XNPT IS IN REGION I
C
      IF(.NOT.(XNPTDM(I) .LT. XH)) GO TO 100
          R1=R0-R0*XNPTDM(I)/XH                          ! EQUATION (3-87)
          R2=R2SVDM(I)
          N=(R-R2)/(R1-R2)                               ! EQUATION (3-33)
          IF(N .LT. 0.) UBAR=1.                          ! EQUATION (3-89)
          IF(N .GT. 1.) UBAR=1./CPM                      ! EQUATION (3-88)
          IF(0. .LT. N .AND. N .LT. 1.)
     *           UBAR=1./CPM+(1.-1./CPM)
     *           *(1.-N**1.5)**2                         ! EQUATION (3-90)
100   CONTINUE
C
C . . . DETERMINE IF XNPT IS IN REGION III
C
      IF(.NOT.(XNPTDM(I) .GT. XN)) GO TO 110
          R2=R2SVDM(I)
          CALL R3
          IF(R .GT. R2) UBAR=1                           ! EQUATION (3-91)
          IF(R .LE. R2) UBAR=1.-DUC*(1.-1./CPM)
     *                  *(1.-(R/R2)**1.5)**2             ! EQUATION (3-93)
110   CONTINUE
C
C . . . DETERMINE IF XNPT IS IN REGION II
C
      IF(.NOT.(XH .LT. XNPTDM(I) .AND. XNPTDM(I) .LT. XN)) GO TO 120
          R1=0.
          R2=R2SVDM(I)
          N=(R-R2)/(R1-R2)                               ! EQUATION (3-33)
          IF(N .LT. 0.) UBAR1=1.                         ! EQUATION (3-89)
          IF(N .GT. 1.) UBAR1=1./CPM                     ! EQUATION (3-88)
          IF(0. .LT. N .AND. N .LT. 1.)
     *           UBAR1=1./CPM+(1.-1./CPM)
     *           *(1.-N**1.5)**2                         ! EQUATION (3-90)
          DUC=1.
          IF(R .GT. R2) UBAR2=1.                         ! EQUATION (3-94)
          IF(R .LE. R2)
     *           UBAR2=1.-DUC*(1.-1./CPM)
     *        *(1.-(R/R2)**1.5)**2                     ! EQUATION (3-93)
          UBAR=((XNPTDM(I)-XH)/(XN-XH))*UBAR2
     *         +((XN-XNPTDM(I))/(XN-XH))*UBAR1            ! EQUATION (3-95)
C      CALL IOWAIT
      CALL HDCOPY
C
C . . . TERMINATE PLOT
C
      CALL ENDPL(0)
C
C . . . RETURN TO MAINLINE
C
      RETURN
      END
```

```
      SUBROUTINE XVSR2
C
      COMMON /PSCALE/RRR,VH0
C
      REAL YVAL(3)
C
C
      NLINES=3
C
C . . . SET MAXIMUM VALUES FOR X AND Y AXIS
C
      XMAX=50.
      YMAX=8.
C
      IF(.NOT.(RRR .GT. 2.)) GO TO 5
          XMAX=50.*RRR
          YMAX=8.*RRR
5     CONTINUE
C
      XSCALE=XMAX/50.
      YSCALE=YMAX/8.
C
C . . . RE-INITIALIZE PLOTTER
C
      CALL BGNPL(1)
      CALL PHYSOR(1.,1.)
      CALL PAGE(14.,11.)
C
C . . . PLOT AXIS
C
      CALL HEIGHT(0.2)
      CALL COMPLX
      CALL MX1ALF('STAND','$')
      CALL MX2ALF('L/CSTD','@')
      CALL MX3ALF('INSTR','&')
      CALL TITLE(' ',1,
     *   'DISTANCE DOWNWIND FROM TURBINE , X/R&LH1.@D$$',
     *   100,'WAKE BOUNDARY PARAMETERS',
     *   100,10.,8.)
      CALL GRAPH(0.,XMAX/10.,0.,YMAX/8.)
      CALL HEADIN('(@1$). NORMALIZED WAKE BOUNDARIES $',100,1.,1)
C
C . . .
C . . . DRAW ONE LINE EACH FOR WAKE RADIUS, WAKE
C . . . RADIUS PLUS ALTITUDE, AND TWO TIMES WAKE RADIUS
C . . .
C
C . . . OPEN DATA SET FOR EACH LINE
C
      DO 40 I=1,NLINES
          OPEN (UNIT=10,FILE='XVSR2.DAT',ACCESS='SEQIN')
C
C . . . READ FIRST CARD
C
          READ(10,10,END=30) XVAL,(YVAL(J),J=1,NLINES)
```

```
10           FORMAT(50F10.4)
           XPOS=(10./XMAX)*XVAL*XSCALE
           YPOS=(8./YMAX)*YVAL(I)*YSCALE
           CALL STRTPT(XPOS,YPOS)
C
C . . . PLOT ENTIRE DATA SET
C
20           CONTINUE
             XPOS=(10./XMAX)*XVAL*XSCALE
             YPOS=(8./YMAX)*YVAL(I)*YSCALE
             CALL CONNPT(XPOS,YPOS)
             READ(10,10,END=30) XVAL,(YVAL(J),J=1,NLINES)
             GO TO 20
30           CONTINUE
C
C . . . CLOSE DATA SET
C
          CLOSE(UNIT=10)
C
40        CONTINUE
C
C . . . MAKE A HARD COPY AND ERASE SCREEN
C
C        CALL IOWAIT
        CALL HDCOPY
C
C . . . TERMINATE PLOT
C
        CALL ENDPL(1)
C
C . . . RETURN TO MAIN ROUTINE
C
        RETURN
        END
```

```
      SUBROUTINE XVSDUC
C
      COMMON /PSCALE/RRR,VH0
C
C . . . SET MAXIMUM VALUES FOR X AND Y AXIS
C
      XMAX=50.
      YMAX=1.
C
      IF(.NOT.(RRR .GT. 2.)) GO TO 5
         XMAX=50.*RRR
5     CONTINUE
C
      IF(.NOT.(VH0 .GT. 1.)) GO TO 6
         YMAX=1.*VH0
6     CONTINUE
C
      XSCALE=XMAX/50.
      YSCALE=YMAX/1.
C
C . . . OPEN DATA SET
C
      OPEN (UNIT=11,FILE='XVSDUC.DAT',ACCESS='SEQIN')
C
C . . . RE-INITIALIZE PLOTTER
C
      CALL BGNPL(1)
      CALL PHYSOR(1.,1.)
      CALL PAGE(14.,11.)
C
C . . . PLOT AXIS
C
      CALL HEIGHT(0.2)
      CALL COMPLX
      CALL MX1ALF('STAND','$')
      CALL MX2ALF('L/CSTD','@')
      CALL MX3ALF('MATHEMATIC','%')
      CALL MX4ALF('INSTR','&')
      CALL Z1USE('$(U&LH1.%O&LXHX$-U&LH1.@C&LXHX$)/$',100)
      CALL Z2USE('$(U&LH1.%O&LXHX$-U&LH1.@O&LXHX$)$',100)
      CALL TITLE(' ',1,
     *  'DISTANCE DOWNWIND FROM TURBINE , X/R&LH1.@D$$',
     *   100,'WIND SPEED DEFICIT &Z1&Z2$',
     *   100,10.,5.)
      CALL GRAPH(0.,XMAX/10.,0.,YMAX/5.)
      CALL HEADIN('(@2$). NORMALIZED WIND SPEED DEFICITS',100,1.,2)
      CALL HEADIN('AT THE WAKE CENTERS',100,1.,2)
C
C . . . READ FIRST CARD
C
      READ(11,10,END=30) XVAL,YVAL
10    FORMAT(2F10.4)
         XPOS=(10./XMAX)*XVAL*XSCALE
         YPOS=(5./YMAX)*YVAL*YSCALE
      CALL STRTPT(XPOS,YPOS)
```

```
C
C . . . PLOT ENTIRE DATA SET
C
20      CONTINUE
            XPOS=(10./XMAX)*XVAL*XSCALE
            YPOS=(5./YMAX)*YVAL*YSCALE
            CALL CONNPT(XPOS,YPOS)
            READ(11,10,END=30) XVAL,YVAL
            GO TO 20
30      CONTINUE
C
C . . . MAKE HARD COPY THEN ERASE SCREEN
C
C        CALL IOWAIT
        CALL HDCOPY
C
C . . . CLOSE DATA SET
C
        CLOSE(UNIT=11)
C
C . . . TERMINATE PLOT
C
        CALL ENDPL(0)
C
C . . . RETURN TO MAIN ROUTINE
C
        RETURN
        END
```

```
      SUBROUTINE XVSUB
C
      COMMON /PSCALE/RRR,VH0
C
C . . . SET MAXIMUM UALUES FOR X AND Y AXIS
C
      XMAX=50.
      YMAX=1.
C
      IF(.NOT.(RRR .GT. 2.)) GO TO 5
         XMAX=50.*RRR
5     CONTINUE
C
      IF(.NOT.(VH0 .GT. 1.)) GO TO 6
         YMAX=1.*VH0
6     CONTINUE
C
      XSCALE=XMAX/50.
      YSCALE=YMAX/1.
C
C . . . OPEN DATA SET
C
      OPEN (UNIT=12,FILE='XVSUB.DAT',ACCESS='SEQIN')
C
C . . . RE-INITIALIZE PLOTTER
C
      CALL BGNPL(1)
      CALL PHYSOR(1.,1.)
      CALL PAGE(14.,11.)
C
C . . . PLOT AXIS
C
      CALL HEIGHT(0.2)
      CALL COMPLX
      CALL MX1ALF('STAND','$')
      CALL MX2ALF('L/CSTD','@')
      CALL MX3ALF('INSTR','&')
      CALL MX4ALF('MATHEMATIC','%')
      CALL Z1USE('$U&LH1.@C&LXHX$/U&LH1.X0$$',100)
      CALL TITLE(' ',1,
     *  'DISTANCE DOUNUIND FROM TURBINE , X/R&LH1.@D$$',
     *   100,'UIND SPEED AT UAKE CENTER, &Z1$',100,10.,5.)
      CALL GRAPH(0.,XMAX/10.,0.,YMAX/5.)
      CALL HEADIN('(@3$). NORMALIZED UIND SPEED$',100,1.,2)
      CALL HEADIN('AT THE UAKE CENTER$',100,1.,2)
C
C . . . READ FIRST CARD
C
      READ(12,10,END=30) XUAL,YUAL
10    FORMAT(2F10.4)
         XPOS=(10./XMAX)*XUAL*XSCALE
         YPOS=(5./YMAX)*YUAL*YSCALE
      CALL STRTPT(XPOS,YPOS)
C
C . . . PLOT ENTIRE DATA SET
```

```
C
20      CONTINUE
            XPOS=(10./XMAX)*XVAL*XSCALE
            YPOS=(6./YMAX)*YVAL*YSCALE
            CALL CONNPT(XPOS,YPOS)
            READ(12,10,END=30) XVAL,YVAL
            GO TO 20
30      CONTINUE
C
C . . . MAKE A HARD COPY THEN ERASE SCREEN
C
C        CALL IOWAIT
        CALL HDCOPY
C
C . . . CLOSE DATA SET
C
        CLOSE(UNIT=12)
C
C . . . TERMINATE PLOT
C
        CALL ENDPL(0)
C
C . . . RETURN TO MAIN ROUTINE
C
        RETURN
        END
```

```
      SUBROUTINE XVSPR
C
      COMMON /PSCALE/RRR,UH0
C
C
C . . . SET MAXIMUM VALUES FOR X AND Y AXIS
C
      XMAX=50.
      YMAX=1.
C
      IF(.NOT.(RRR .GT. 2)) GO TO 5
        XMAX=50.*RRR
5     CONTINUE
C
      IF(.NOT.(UH0 .GT. 1)) GO TO 6
        YMAX=1.*UH0
6     CONTINUE
C
      XSCALE=XMAX/50.
      YSCALE=YMAX/1.
C
C . . . OPEN DATA SET
C
      OPEN (UNIT=14,FILE='XVSPR.DAT',ACCESS='SEQIN')
C
C . . . RE-INITIALIZE PLOTTER
C
      CALL BGNPL(1)
      CALL PHYSOR(1.,1.)
      CALL PAGE(14.,11.)
C
C . . . PLOT AXIS
C
      CALL HEIGHT(0.2)
      CALL COMPLX
      CALL MX1ALF('STAND','$')
      CALL MX2ALF('L/CSTD','@')
      CALL MX3ALF('MATHEMATIC','%')
      CALL MX4ALF('INSTR','&')
      CALL TITLE(' ',1,
     *  'DISTANCE DOWNWIND FROM TURBINE , X/R&LH1.@D$$',
     *   100,'POWER FACTOR, P/P&LH1.%O$',100,10.,5.)
      CALL GRAPH(0.,XMAX/10.,0.,YMAX/5.)
      CALL HEADIN('(@4$). TURBINE POWER FACTORS',100,1.,2)
      CALL HEADIN('FOR AN UNBOUNDED WAKES',100,1.,2)
C
C . . . READ FIRST CARD
C
      READ(14,10,END=30) XVAL,YVAL
10    FORMAT(2F10.4)
        XPOS=(10./XMAX)*XVAL*XSCALE
        YPOS=(5./YMAX)*YVAL*YSCALE
      CALL STRTPT(XPOS,YPOS)
C
C . . . PLOT ENTIRE DATA SET
```

```
C
20      CONTINUE
            XPOS=(10./XMAX)*XVAL*XSCALE
            YPOS=(5./YMAX)*YVAL*YSCALE
            CALL CONNPT(XPOS,YPOS)
            READ(14,10,END=30) XVAL,YVAL
            GO TO 20
30      CONTINUE
C
C . . . MAKE HARD COPY THEN ERASE SCREEN
C
C        CALL IOWAIT
        CALL HDCOPY
C
C . . . CLOSE DATA SET
C
        CLOSE(UNIT=14)
C
C . . . TERMINATE PLOT
C
        CALL ENDPL(0)
C
C . . . RETURN TO MAIN ROUTINE
C
        RETURN
        END
```

```
      SUBROUTINE VVSUBR(IUNIT,UFILE,XNPTDM,I)
C
      COMMON /PSCALE/RRR,VH0
C
      DOUBLE PRECISION UFILE
C
      REAL XNPTDM(20)
C
      INTEGER WORK(13),ITITLE(13),PLTNUM
C
      LOGICAL NEWLIN
C
      DATA FLAG/-99./
C
C . . . SET PLOT NUMBER FOR TITLE ANNOTATION
C
      PLTNUM=I+4
C
C . . . SET MAXIMUM VALUES FOR X AND Y AXIS
C
      XMAX=6.
      YMAX=1.2
C
      IF(.NOT.(RRR .GT. 2.)) GO TO 1
          XMAX=6.*RRR
1     CONTINUE
C
      IF(.NOT.(VH0 .GT. 1.)) GO TO 2
          YMAX=1.2*VH0
2     CONTINUE
C
      XSCALE=XMAX/6.
      YSCALE=YMAX/1.2
C
C . . . OPEN DATA SET
C
      OPEN(UNIT=IUNIT,FILE=UFILE,ACCESS='SEQIN')
C
C . . . 'CREATE' TITLE FOR PLOT
C
      ENCODE(65,5,WORK) PLTNUM,XNPTDM(I)
5     FORMAT('(',I2,'). LATERAL WIND SPEED PROFILE
     *   'AT X/R&LH1.0D&LXHX$ =',F7.2,'$')
C
      DECODE(65,6,WORK) ITITLE
6     FORMAT(15A5)
C
C . . .RE-INITIALIZE PLOTTER
C
      CALL BGNPL(1)
      CALL PHYSOR(1.,1.)
      CALL PAGE(9.,9.)
C
C . . . PLOT AXIS
C
```

```
      CALL HEIGHT(0.2)
      CALL COMPLX
      CALL MX1ALF('STAND','$')
      CALL MX2ALF('L/CSTD','@')
      CALL MX3ALF('MATHEMATIC','%')
      CALL MX4ALF('INSTR','&')
      CALL TITLE(' ',1,'LATERAL COORDINATE , Y/R&LH1.@D$$',100,
     *            'WIND SPEED, U/U&LH1.%O$$',100,6.,6.)
      CALL GRAPH(0.,XMAX/6.,0.,YMAX/6.)
      CALL HEADIN(ITITLE(1),35,1.,2)
      CALL HEADIN(ITITLE(8),100,1.,2)
C
C . . . READ FIRST RECORD
C
      READ(IUNIT,10,END=50) YVAL,UBAR
10    FORMAT(2F10.4)
C
C . . .
C . . . PLOT DATA SET
C . . .
C
15    CONTINUE
C
          IF(.NOT.(YVAL .EQ. FLAG)) GO TO 20
              NEWLIN=.TRUE.
20        CONTINUE
C
          IF(.NOT. NEWLIN) GO TO 30
              READ(IUNIT,10,END=50)YVAL,UBAR
              XPOS=YVAL*(6./XMAX)*XSCALE
              YPOS=UBAR*(6./YMAX)*YSCALE
              CALL STRTPT(XPOS,YPOS)
              NEWLIN=.FALSE.
30        CONTINUE
C
          IF(.NOT.(.NOT. NEWLIN)) GO TO 40
              XPOS=YVAL*(6./XMAX)*XSCALE
              YPOS=UBAR*(6./YMAX)*YSCALE
              IF(.NOT.(XPOS .LT. 10.)) GO TO 35
                  CALL CONNPT(XPOS,YPOS)
35            CONTINUE
40        CONTINUE
C
C
          READ(IUNIT,10,END=50)YVAL,UBAR
          GO TO 15
C
50    CONTINUE
C
C . . . CLOSE FILES
C
      CLOSE(UNIT=IUNIT,DISPOSE='SAVE')
C
C . . . MAKE A HARD COPY AND ERASE SCREEN
C
120   CONTINUE
C
C . . . RETURN TO MAIN LINE
C
      RETURN
      END
```

```
$
        SUBROUTINE UVSZ(IUNIT,FILE,AH,J)
C
        COMMON /PSCALE/RRR,VH0
C
        REAL XNPTDM(20)
C
        INTEGER WORK(12),ITITLE(12),PLTNUM
C
        LOGICAL NEWLIN
C
        DATA FLAG/-99./
C
C . . . SET PLOT NUMBER FOR TITLE
C
        PLTNUM=4*J+1
C
C . . . SET MAXIMUM VALUES FOR X AND Y AXIS
C
        XMAX=1.2
        YMAX=5.
C
        IF(.NOT.(RRR .GT. 2)) GO TO 5
           YMAX=5.*RRR
5       CONTINUE
C
        IF(.NOT.(VH0 .GT. 1.)) GO TO 6
           XMAX=1.2*VH0
6       CONTINUE
C
        XSCALE=XMAX/1.2
        YSCALE=YMAX/5.
C
C . . . OPEN DATA SET
C
        OPEN(UNIT=IUNIT,FILE=FILE,ACCESS='SEQIN',
     *       DEVICE='DSK')
C
C . . . PRODUCE TITLE
C
        ENCODE(60,7,WORK) PLTNUM
7       FORMAT('(',I2,'). VERTICAL WIND SPEED PROFILES
     *    'AT WAKE CENTERS')
C
        DECODE(60,8,WORK)ITITLE
8       FORMAT(12A5)
C
C . . .RE-INITIALIZE PLOTTER
C
        CALL BGNPL(1)
        CALL PHYSOR(1.,1.)
        CALL PAGE(8.,8.)
C
C . . . PLOT AXIS
C
        CALL HEIGHT(0.2)
*
```

```
      CALL COMPLX
      CALL MX1ALF('STAND','$')
      CALL MX2ALF('L/CSTD','@')
      CALL MX3ALF('MATHEMATIC','%')
      CALL MX4ALF('INSTR','&')
      CALL TITLE(' ',1,'WIND SPEED, U/U&LH1.%O$$',100,
     *   'ALTITUDE, Z/R&LH1.@D$$',100,6.,5.)
      CALL GRAPH(0.,XMAX/6.,0.,YMAX/5.)
      CALL HEADIN(ITITLE(1),35,1.,2)
      CALL HEADIN(ITITLE(8),100,1.,2)
C
C . . . READ FIRST RECORD
C
      READ(IUNIT,10,END=50) UVAL,ZVAL
10    FORMAT(2F10.4)
C
C . . .
C . . . PLOT DATA SET
C . . .
C
15    CONTINUE
C
          IF(.NOT.(UVAL .EQ. FLAG)) GO TO 20
              NEWLIN=.TRUE.
20        CONTINUE
C
          IF(.NOT. NEWLIN) GO TO 30
              READ(IUNIT,10,END=50)UVAL,ZVAL
              XPOS=UVAL*(6./XMAX)*XSCALE
              YPOS=ZVAL*(5./YMAX)*YSCALE
              CALL STRTPT(XPOS,YPOS)
              NEWLIN=.FALSE.
30        CONTINUE
C
          IF(.NOT.(.NOT. NEWLIN)) GO TO 40
              XPOS=UVAL*(6./XMAX)*XSCALE
              YPOS=ZVAL*(5./YMAX)*YSCALE
              IF(.NOT.(XPOS .LT. 10.)) GO TO 35
                  CALL CONNPT(XPOS,YPOS)
35            CONTINUE
40        CONTINUE
C
C
          READ(IUNIT,10,END=50)UVAL,ZVAL
          GO TO 15
C
50    CONTINUE
C
C . . . DRAW A STRAIGHT LINE AT Z=AH
C
      XPOS=0.
      YPOS=(5./YMAX)*AH*YSCALE
      CALL STRTPT(XPOS,YPOS)
      XPOS=6.
      CALL CONNPT(XPOS,YPOS)
```

```
C
C . . . DRAW A DASH AT Z=AH+1
C
        XPOS=0.
        YPOS=(5./YMAX)*(AH+1)*YSCALE
        CALL STRTPT(XPOS,YPOS)
        XPOS=0.5
        CALL CONNPT(XPOS,YPOS)
C
C . . . DRAW A DASH AT Z=AH-1
C
        XPOS=0.
        YPOS=(5./YMAX)*(AH-1)*YSCALE
        CALL STRTPT(XPOS,YPOS)
        XPOS=0.5
        CALL CONNPT(XPOS,YPOS)
C
C . . . CLOSE FILES
C
        CLOSE(UNIT=IUNIT)
C
C . . . MAKE A HARD COPY AND ERASE SCREEN
C
C        CALL IOWAIT
        CALL HDCOPY
C
C . . . TERMINATE PLOT
C
        CALL ENDPL(0)
C
C . . . RETURN TO MAINLINE
C
        RETURN
        END
```

Appendix C

WAKE CALCULATIONS ON THE TI-59 CALCULATOR

This appendix contains a description of a version of the wake model for the Texas Instruments TI-59 programmable calculator. A Texas Instruments PC-100C print cradle is used with the calculator. Mathematically, the model is identical to the model presented in this report and impletmented as a FORTRAN computer program. There are some limitations associated with the calculator version of the model. First, no plots are produced. All output is in numerical form and printed on the PC-100C printer. Second, all units of length are normalized by the rotor radius, and all units of wind speed are normalized by the free stream wind speed. There are no options for output in physical units. Third, atmospheric turbulence must be input as the standard deviation of wind direction. There is no option for input as a Pasquill class.

There are two programs described in this appendix. The first program calculates values of downwind distance, x, wake radius, r_2, normalized wind speed deficit at the wake center, Δu_c, normalized wind speed at the wake center, u_c, and turbine power factor, $\overline{P}$. The second program generates numbers for wind speed profiles for Region III.

It is assumed that the user of these programs is familiar with the TI-59 programmable calculator and is reasonably proficient in programming the calculator. The TI-59 calculator was chosen because it is widely used in scientific institutions. The wake program will not fit on the TI-58 calculator. The program for wind speed profiles will fit on the TI-58. Program listings are given on the following pages.

Wake Program

For the wake program, the calculator must be properly partitioned. Input of 3 OP17 will properly partition the calculator with 720 program steps and 30 memory locations.

The inputs to the program are the initial wind speed ratio, m, and the standard deviation of ambient wind direction, σ_θ. To run the program, enter m, and press A. The input value of m will be printed. When 0.02 appears on the display, enter σ_θ, and press B. The value of σ_θ will be printed. A value of σ_θ = -1000 should be used to obtain the Abramovich solution (i.e., to make $d\sigma/dx = 0$ in equation (3-20)). Output consists of sets of five numbers: downwind location, x; wake radius, r_2; normalized wind speed deficit at the wake center,

Δu_c; normalized wind speed at the wake center, u_c; and turbine power factor, $\overline{P}$. The first set of numbers is for the initial wake (i.e., x = 0). The second set of numbers is for the end of Region I (i.e., $x = x_H$). The third set of numbers is for the end of Region II (i.e., $x = x_N$). The following sets of numbers are printed during the numerical integration in Region III. Since the turbine power factor is undefined in Regions I and II, it does not appear with the first three sets of numbers. Table C-1 shows sample output. The program terminates if x>60. The user may terminate the program at any time by pressing R/S.

The parameters stored in the 30 memory locations are shown in Table C-2. This information is not necessary for running the program but is given for users who may desire to modify the program. A TI-59 listing of the program is given below. The description of the program is given adjacent to the program listing.

Input m	000	76	LBL
	001	11	A
	002	42	STO
	003	02	02
Print m	004	98	ADV
	005	99	PRT
Set value of x at which integration of Region III ends	006	06	6
	007	00	0
	008	32	X:T
For numerical integration in Region III, set $\Delta r = 0.02$	009	93	.
	010	00	0
	011	02	2
	012	42	STO
	013	19	19
Input σ_θ	014	91	R/S
	015	76	LBL
	016	12	B
Print σ_θ	017	99	PRT

Calculate α from equations (3-20) and (3-28) and store in location 01	018	53	(
	019	24	CE
	020	65	×
	021	93	.
	022	00	0
	023	08	8
	024	54	)
	025	22	INV
	026	23	LNX
	027	65	×
	028	93	.
	029	00	0
	030	03	3
	031	01	1
	032	65	×
	033	01	1
	034	93	.
	035	03	9
	036	09	7
	037	95	=
	038	42	STO
	039	01	01
Print x = 0	040	98	ADV
	041	00	0
	042	99	PRT

Calculate r_0 from equation (3-18) and store in location 03	043	53	(
	044	53	(
	045	43	RCL
	046	02	02
	047	85	+
	048	01	1
	049	54	)
	050	55	÷
	051	02	2
	052	54	)
	053	34	√X
	054	95	=
	055	42	STO
	056	03	03
Print r_0	057	99	PRT
Print Δu_c	058	01	1
	059	99	PRT
Print u_c	060	43	RCL
	061	02	02
	062	35	1/X
	063	99	PRT
Calculate $\sqrt{0.214+0.144m}$ and store in location 05	064	25	CLR
	065	53	(
	066	93	.
	067	02	2
	068	01	1
	069	04	4
	070	85	+
	071	93	.
	072	01	1
	073	04	4
	074	04	4
	075	65	×
	076	43	RCL
	077	02	02
	078	54	)
	079	34	√X
	080	95	=
	081	42	STO
	082	05	05
Calculate r_{21} from equation (3-34) and store in location 04	083	43	RCL
	084	03	03
	085	55	÷
	086	43	RCL
	087	05	05
	088	95	=
	089	42	STO
	090	04	04
Calculate $(x_H)_m$ from equation (3-35) and store in location 06	091	43	RCL
	092	03	03
	093	65	×
	094	53	(
	095	01	1
	096	85	+
	097	43	RCL
	098	02	02
	099	54	)
	100	55	÷
	101	53	(
	102	93	.
	103	02	2
	104	07	7
	105	65	×
	106	53	(
	107	43	RCL
	108	02	02
	109	75	-
	110	01	1
	111	54	)
	112	65	×
	113	43	RCL
	114	05	05
	115	54	)
	116	95	=
	117	42	STO
	118	06	06

Description	Step	Code	Key
Calculate x_H from equation (3-39) and store in location 06	119	43	RCL
	120	04	04
	121	55	÷
	122	53	(
	123	53	(
	124	43	RCL
	125	04	04
	126	55	÷
	127	43	RCL
	128	06	06
	129	54	)
	130	33	X²
	131	85	+
	132	53	(
	133	02	2
	134	65	×
	135	43	RCL
	136	01	01
	137	54	)
	138	33	X²
	139	54	)
	140	34	√X
	141	95	=
	142	42	STO
	143	06	06
Print x_H	144	98	ADV
	145	99	PRT
Print r_{21}	146	43	RCL
	147	04	04
	148	99	PRT
Print Δu_c	149	01	1
	150	99	PRT
Print $u_c = 1/m$	151	43	RCL
	152	02	02
	153	35	1/X
	154	99	PRT
	155	25	CLR
Calculate $\sqrt{0.134+.124m}$ and store in location 07	156	53	(
	157	93	.
	158	01	1
	159	03	3
	160	04	4
	161	85	+
	162	93	.
	163	01	1
	164	02	2
	165	04	4
	166	65	×
	167	43	RCL
	168	02	02
	169	54	)
	170	34	√X
	171	95	=
	172	42	STO
	173	07	07
Calculate n from equation (3-48) and store in location 08	174	43	RCL
	175	05	05
	176	65	×
	177	53	(
	178	01	1
	179	75	-
	180	43	RCL
	181	07	07
	182	54	)
	183	55	÷
	184	53	(
	185	53	(
	186	01	1
	187	75	-
	188	43	RCL
	189	05	05
	190	54	)
	191	65	×
	192	43	RCL
	193	07	07
	194	54	)
	195	95	=
	196	42	STO
	197	08	08

Calculate x_N from equation (3-47) and store in location 09	198	43	RCL
	199	06	06
	200	65	×
	201	43	RCL
	202	08	08
	203	95	=
	204	42	STO
	205	09	09
Print x_N	206	98	ADV
	207	99	PRT
Calcualte r_{22} from equation (3-50) and store in location 10	208	43	RCL
	209	03	03
	210	85	+
	211	43	RCL
	212	08	08
	213	65	×
	214	53	(
	215	43	RCL
	216	04	04
	217	75	-
	218	43	RCL
	219	03	03
	220	54	)
	221	95	=
	222	42	STO
	223	10	10
Print r_{22}	224	99	PRT
Store r_{22} in locations 11 and 14 as the initial radius for R. III	225	42	STO
	226	11	11
	227	42	STO
	228	14	14
Print Δu_c	229	01	1
	230	99	PRT
Print u_c = 1/m	231	43	RCL
	232	02	02
	233	35	1/X
	234	99	PRT
Calculate Δu_c from Subroutine A' and store in location 12	235	25	CLR
	236	71	SBR
	237	16	A'
	238	42	STO
	239	12	12
Calculate $(dr/dx)_m$ from Subroutine B' and store in location 13	240	71	SBR
	241	17	B'
	242	42	STO
	243	13	13

Label D	244	76	LBL
	245	14	D
Calculate r' and store in location 15	246	43	RCL
	247	11	11
	248	85	+
	249	43	RCL
	250	19	19
	251	95	=
	252	42	STO
	253	15	15
Store r' in location 14 for Subroutine A'	254	66	PAU
	255	42	STO
	256	14	14
Call Subroutine A' for Δu_c'	257	71	SBR
	258	16	A'
Call Subroutine B' for $(dr/dx)_m$'	259	71	SBR
	260	17	B'
Store $(dr/dx)_m$' in location 17	261	42	STO
	262	17	17
Calculate $(dr/dx)_e$ and store in location 18	263	53	(
	264	53	(
	265	53	(
	266	43	RCL
	267	13	13
	268	85	+
	269	43	RCL
	270	17	17
	271	54	)
	272	55	÷
	273	02	2
	274	54	)
	275	33	X^2
	276	85	+
	277	43	RCL
	278	01	01
	279	33	X^2
	280	54	)
	281	34	√X
	282	95	=
	283	42	STO
	284	18	18

Description	Step	Code	Key
Calculate x' from equation (3-62) and store in location 09	285	43	RCL
	286	09	09
	287	85	+
	288	43	RCL
	289	19	19
	290	55	÷
	291	43	RCL
	292	18	18
	293	95	=
	294	42	STO
	295	09	09
Print x'	296	98	ADV
	297	99	PRT
Print r'	298	43	RCL
	299	15	15
	300	99	PRT
Print Δu_c'	301	43	RCL
	302	16	16
	303	99	PRT
Calculate u_c' and print	304	01	1
	305	75	-
	306	43	RCL
	307	16	16
	308	65	×
	309	53	(
	310	01	1
	311	75	-
	312	43	RCL
	313	02	02
	314	35	1/X
	315	54	)
	316	95	=
	317	99	PRT
Calculate B from equation (3-75) and store in location 20	318	43	RCL
	319	16	16
	320	65	×
	321	53	(
	322	01	1
	323	75	-
	324	43	RCL
	325	02	02
	326	35	1/X
	327	54	)
	328	95	=
	329	42	STO
	330	20	20
Calculate first coefficient of equation (3-78) and store in location 21	331	53	(
	332	01	1
	333	75	-
	334	03	3
	335	65	×
	336	43	RCL
	337	20	20
	338	85	+
	339	03	3
	340	65	×
	341	43	RCL
	342	20	20
	343	33	X^2
	344	75	-
	345	43	RCL
	346	20	20
	347	45	Y^X
	348	03	3
	349	54	)
	350	55	÷
	351	02	2
	352	95	=
	353	42	STO
	354	21	21
Calculate second coefficient of equation (3-78) and store in location 22	355	53	(
	356	06	6
	357	65	×
	358	43	RCL
	359	20	20
	360	75	-
	361	01	1
	362	02	2
	363	65	×
	364	43	RCL
	365	20	20
	366	33	X^2
	367	85	+
	368	06	6
	369	65	×
	370	43	RCL
	371	20	20
	372	45	Y^X
	373	03	3
	374	54	)
	375	55	÷
	376	03	3
	377	93	.
	378	05	5
	379	95	=
	380	42	STO
	381	22	22

Description	Step	Code	Key
Calculate third coefficient of equation (3-78) and store in location 23	382	53	(
	383	03	3
	384	94	+/-
	385	65	×
	386	43	RCL
	387	20	20
	388	85	+
	389	01	1
	390	08	8
	391	65	×
	392	43	RCL
	393	20	20
	394	33	X²
	395	75	-
	396	01	1
	397	05	5
	398	65	×
	399	43	RCL
	400	20	20
	401	45	Y^X
	402	03	3
	403	54	)
	404	55	÷
	405	05	5
	406	95	=
	407	42	STO
	408	23	23
Calculate fourth coefficient of equation (3-78) and store in location 24	409	53	(
	410	01	1
	411	02	2
	412	94	+/-
	413	65	×
	414	43	RCL
	415	20	20
	416	33	X²
	417	85	+
	418	02	2
	419	00	0
	420	65	×
	421	43	RCL
	422	20	20
	423	45	Y^X
	424	03	3
	425	54	)
	426	55	÷
	427	06	6
	428	93	.
	429	05	5
	430	95	=
	431	42	STO
	432	24	24
Calculate fifth coefficient of equation (3-78) and store in location 25	433	53	(
	434	03	3
	435	65	×
	436	43	RCL
	437	20	20
	438	33	X²
	439	75	-
	440	01	1
	441	05	5
	442	65	×
	443	43	RCL
	444	20	20
	445	45	Y^X
	446	03	3
	447	54	)
	448	55	÷
	449	08	8
	450	95	=
	451	42	STO
	452	25	25
Calculate sixth coefficient of equation (3-78) and store in location 26	453	06	6
	454	65	×
	455	43	RCL
	456	20	20
	457	45	Y^X
	458	03	3
	459	55	÷
	460	09	9
	461	93	.
	462	05	5
	463	95	=
	464	42	STO
	465	26	26
Calculate seventh coefficient of equation (3-78) and store in location 27	466	01	1
	467	94	+/-
	468	65	×
	469	43	RCL
	470	20	20
	471	45	Y^X
	472	03	3
	473	55	÷
	474	01	1
	475	01	1
	476	95	=
	477	42	STO
	478	27	27

Description	Step	Code	Key
Calculate r_∞ from equation (3-72) and store in location 28	479	53	(
	480	53	(
	481	43	RCL
	482	02	02
	483	85	+
	484	01	1
	485	54	)
	486	55	÷
	487	53	(
	488	02	2
	489	65	×
	490	43	RCL
	491	02	02
	492	54	)
	493	54	)
	494	34	√X
	495	95	=
	496	42	STO
	497	28	28
Calculate r_∞/r_2 and store in location 29	498	24	CE
	499	55	÷
	500	43	RCL
	501	15	15
	502	95	=
	503	42	STO
	504	29	29
Calculate $\overline{P}$ from equation (3-78)	505	02	2
	506	65	×
	507	53	(
	508	43	RCL
	509	21	21
	510	85	+
	511	43	RCL
	512	22	22
	513	65	×
	514	43	RCL
	515	29	29
	516	45	Y^X
	517	01	1
	518	93	.
	519	05	5
	520	85	+

Description	Step	Code	Key
Calculate $\overline{P}$ from equation (3-78) (concluded)	521	43	RCL
	522	23	23
	523	65	×
	524	43	RCL
	525	29	29
	526	45	Y^X
	527	03	3
	528	85	+
	529	43	RCL
	530	24	24
	531	65	×
	532	43	RCL
	533	29	29
	534	45	Y^X
	535	04	4
	536	93	.
	537	05	5
	538	85	+
	539	43	RCL
	540	25	25
	541	65	×
	542	43	RCL
	543	29	29
	544	45	Y^X
	545	06	6
	546	85	+
	547	43	RCL
	548	26	26
	549	65	×
	550	43	RCL
	551	29	29
	552	45	Y^X
	553	07	7
	554	93	.
	555	05	5
	556	85	+
	557	43	RCL
	558	27	27
	559	65	×
	560	43	RCL
	561	29	29
	562	45	Y^X
	563	09	9
	564	54	)
	565	95	=
Print $\overline{P}$	566	99	PRT

Set r = r'	567	43	RCL
	568	15	15
	569	42	STO
	570	11	11
Set $\Delta u_c = \Delta u_c'$	571	43	RCL
	572	16	16
	573	42	STO
	574	12	12
Set $(dr/dx)_m = (dr/dx)_m'$	575	43	RCL
	576	17	17
	577	42	STO
	578	13	13
If x < 60, GO TO D	579	43	RCL
	580	09	09
	581	22	INV
	582	77	GE
	583	14	D
Stop	584	91	R/S
Subroutine A' to calculate Δu_c by equation (3-58)	585	76	LBL
	586	16	A'
	587	03	3
	588	93	.
	589	07	7
	590	03	3
	591	65	×
	592	53	(
	593	93	.
	594	02	2
	595	05	5
	596	08	8
	597	65	×
	598	43	RCL
	599	02	02
	600	55	÷
Subroutine A' (concluded)	601	53	(
	602	43	RCL
	603	02	02
	604	75	-
	605	01	1
	606	54	)
	607	75	-
	608	53	(
	609	24	CE
	610	33	X^2
	611	75	-
	612	93	.
	613	05	5
	614	03	3
	615	06	6
	616	65	×
	617	43	RCL
	618	03	03
	619	33	X^2
	620	55	÷
	621	53	(
	622	43	RCL
	623	14	14
	624	33	X^2
	625	65	×
	626	53	(
	627	43	RCL
	628	02	02
	629	75	-
	630	01	1
	631	54	)
	632	54	)
	633	54	)
	634	34	$\sqrt{X}$
	635	54	)
	636	95	=
	637	42	STO
	638	16	16
	639	92	RTN

	640	76	LBL
	641	17	B'
	642	93	.
	643	02	2
	644	07	7
	645	55	÷
	646	53	(
	647	02	2
	648	65	×
Subroutine B' to calculate $(dr/dx)_m$ by equation (3-60)	649	43	RCL
	650	02	02
	651	55	÷
	652	53	(
	653	53	(
	654	43	RCL
	655	02	02
	656	75	-
	657	01	1
	658	54	)
	659	65	×
	660	43	RCL
	661	16	16
	662	54	)
	663	75	-
	664	01	1
	665	54	)
	666	95	=
	667	92	RTN

Table C-1

OUTPUT OF WAKE PROGRAM FOR TI-59 CALCULATOR

Description	Output	Symbol
Input value of m	2.	m
Input value of σ_θ	10.	σ_θ
Initial wake	0.	$x=0$
	1.224744871	r_0
	1.	Δu_c
	0.5	u_c
Wake at end of Region I	8.15901383	x_H
	1.728597064	r_{21}
	1.	Δu_c
	0.5	u_c
Wake at end of Region II	12.25580717	x_N
	1.981590667	r_{22}
	1.	Δu_c
	0.5	u_c
First point in numerical integration in Region III	12.40933542	x
	2.001590667	r_2
	.9695134231	Δu_c
	.5152432884	u_c
	.2916912548	$\overline{P}$
Second point in numerical integration in Region III	12.5656277	x
	2.021590667	r_2
	.9411585052	Δu_c
	.5294207474	u_c
	.3019249019	$\overline{P}$
Third point in numerical integration in Region III	12.72450863	x
	2.041590667	r_2
	.9143937763	Δu_c
	.5428031119	u_c
	.3118966655	$\overline{P}$

Table C-2

STORAGE LOCATIONS USED FOR
WAKE PROGRAM FOR TI-59 CALCULATOR

Storage location	Symbol	Definition
01	α	Wake growth rate due to ambient turbulence
02	m	Initial wind speed ratio, U_∞/U_0
03	r_0	Initial wake radius
04	r_{21}	Wake radius at the end of Region I
05		$\sqrt{0.214+0.144m}$
06	$(x_H)_m$ and x_H	Downwind extent of Region I from Abramovich solution and downwind extent of Region I
07		$\sqrt{0.134+0.124m}$
08	n	X_N/X_H
09	x	Downwind coordinate
10	r_{22}	Wake radius at end of Region II
11	r_2	Wake radius
12	Δu_c	Wind speed deficit parameter
13	$(dr/dx)_m$	Wake growth rate due to mechanical turbulence for wake radius, r_2
14	r_2	Wake radius used by Subroutine A'
15	r_2'	Incremented value of wake radius
16	$\Delta u_c'$	Wind speed deficit parameter for wake radius, r'
17	$(dr/dx)_m$	Wake growth rate due to mechanical turbulence for wake radius, r_2'
18	$(dr/dx)_e$	Effective growth rate of the wake radius
19	Δr	Increment in wake radius for integration
20	B	$\Delta u_c(1-1/m)$
21		First coefficient for equation (3-78)
22		Second coefficient for equation (3-78)

Table C-2

STORAGE LOCATIONS USED FOR
WAKE PROGRAM FOR TI-59 CALCULATOR (concluded)

Storage location	Symbol	Definition
23		Third coefficient for equation (3-78)
24		Fourth coefficient for equation (3-78)
25		Fifth coefficient for equation (3-78)
26		Sixth coefficient for equation (3-78)
27		Seventh coefficient for equation (3-78)
28	r_∞	Radius of stream tube in the free stream flow
29	r_∞/r_2	Ratio of r_∞ for the downwind turbine to the wake radius of the wake of the upwind turbine

Wind Speed Profiles

The program for wind speed profiles generates the wind speed in the wake for the lateral wind speed profiles or for the vertical wind speed profile. The inputs to the program are the initial wind speed ratio, m; the height of the rotor hub (in rotor radii), h_a; the power law coefficient for the free stream wind speed profile, γ; and the wake radius r_2. The wake radius may be obtained for a given downwind location from the wake program. For horizontal wind speed profiles, the altitude at which the profile is to be calculated, z, is also input. The output of the program is the normalized wind speed in the wake at intervals of 0.1 rotor radii. The wind speed in the wake is normalized by the free stream wind speed at the hub altitude. An input value of $\gamma = 0$ allows the wind speed to be normalized by the wind speed at the altitude of the profile.

To operate the program:

Enter m	Press A
Enter h_a	Press R/S
Enter γ	Press R/S
Enter r_2	Press B

For the lateral wind speed profiles:

Enter z Press C

For the vertical wind speed profile:

Press E

For lateral wind speed profiles at an additional altitude:

Enter new z Press C

To change r_2:

Enter new r_2 Press B

The program will terminate when the point on the profile is in the free stream wind. The last number printed is for the free stream wind. Table C-3 shows a sample output. Table C-4 shows the parameters stored in the memory locations. A TI-59 listing of the program is given on the next page. The description of the program is given adjacent to the program listing.

Input m	000	76	LBL
	001	11	A
	002	42	STO
	003	01	01
Print m	004	99	PRT
	005	91	R/S
Input h_a	006	42	STO
	007	02	02
Print h_a	008	99	PRT
	009	91	R/S
Input γ	010	42	STO
	011	13	13
Print γ	012	99	PRT
Set Δy=0.1	013	93	.
	014	01	1
	015	42	STO
	016	03	03
Put 1 in t register	017	01	1
	018	32	X⇌T
	019	91	R/S
Input r_2	020	76	LBL
	021	12	B
	022	42	STO
	023	04	04
Print r_2	024	98	ADV
	025	98	ADV
	026	99	PRT
Calculate r_0 from equation (3-18) and store in location 05	027	53	(
	028	53	(
	029	43	RCL
	030	01	01
	031	85	+
	032	01	1
	033	54	)
	034	55	÷
	035	02	2
	036	54	)
	037	34	√X
	038	95	=
	039	42	STO
	040	05	05
Calcuate Δu_c from equation (3-58)	041	03	3
	042	93	.
	043	07	7
	044	03	3
	045	65	×
	046	53	(
	047	53	(
	048	93	.
	049	02	2
	050	05	5
	051	08	8
	052	65	×
	053	43	RCL
	054	01	01
	055	55	÷
	056	53	(
	057	43	RCL
	058	01	01
	059	75	-
	060	01	1
	061	54	)
	062	54	)
	063	75	-
	064	53	(
	065	24	CE
	066	33	X²
	067	75	-
	068	93	.
	069	05	5
	070	03	3
	071	06	6
	072	65	×
	073	43	RCL
	074	05	05
	075	33	X²
	076	55	÷
	077	53	(
	078	43	RCL
	079	04	04
	080	33	X²
	081	65	×
	082	53	(
	083	43	RCL
	084	01	01
	085	75	-
	086	01	1
	087	54	)
	088	54	)
	089	54	)
	090	34	√X
	091	54	)
	092	95	=

Store Δu_c in location 06	093	42	STO
	094	06	06
Input z for lateral wind speed profile	095	91	R/S
	096	76	LBL
	097	13	C
	098	42	STO
	099	07	07
Print z	100	98	ADV
	101	98	ADV
	102	99	PRT
Initialize y	103	00	0
	104	42	STO
	105	08	08
Calculate z_v from equation (3-85) and store in location 09	106	76	LBL
	107	17	B'
	108	43	RCL
	109	02	02
	110	75	-
	111	43	RCL
	112	07	07
	113	95	=
	114	42	STO
	115	09	09
Calculate u from Subroutine A' and store in location 10	116	71	SBR
	117	16	A'
	118	42	STO
	119	10	10
Calculate z_v* from equation (3-96) and store in location 09	120	43	RCL
	121	02	02
	122	85	+
	123	43	RCL
	124	07	07
	125	95	=
Calclated u* from Subroutine A' and store in location 11	126	42	STO
	127	09	09
	128	71	SBR
	129	16	A'
Print y	130	42	STO
	131	11	11
Increase y by Δy	132	43	RCL
	133	08	08
	134	98	ADV
	135	99	PRT
	136	43	RCL
	137	03	03
	138	44	SUM
	139	08	08
Calculate U from equation (3-98) and print	140	43	RCL
	141	10	10
	142	65	×
	143	43	RCL
	144	11	11
	145	65	×
	146	53	(
	147	43	RCL
	148	07	07
	149	55	÷
	150	43	RCL
	151	02	02
	152	54	)
	153	45	Y^x
	154	43	RCL
	155	13	13
	156	95	=
	157	99	PRT
Go to B' if not in free stream flow	158	43	RCL
	159	10	10
	160	65	×
	161	43	RCL
	162	11	11
	163	95	=
	164	22	INV
	165	67	EQ
	166	17	B'
Stop	167	91	R/S
Vertical wind speed profile	168	76	LBL
	169	15	E
Initialize z	170	00	0
	171	42	STO
	172	08	08
	173	42	STO
	174	07	07
Calculate z_v from equation (3-85) and store in location 09	175	76	LBL
	176	10	E'
	177	43	RCL
	178	02	02
	179	75	-
	180	43	RCL
	181	07	07
	182	95	=
	183	42	STO
	184	09	09
Calculate u from Subroutine A' and store in location 10	185	71	SBR
	186	16	A'
	187	42	STO
	188	10	10

Description	Step	Code	Key
	189	43	RCL
Calculate z_v* from equation (3-96) and store in location 09	190	02	02
	191	85	+
	192	43	RCL
	193	07	07
	194	95	=
	195	42	STO
	196	09	09
Calculate u* from Subroutine A' and store in location 11	197	71	SBR
	198	16	A'
	199	42	STO
	200	11	11
Print z	201	43	RCL
	202	07	07
	203	98	ADV
	204	99	PRT
Calculate U from equation (3-98) and print	205	43	RCL
	206	10	10
	207	65	×
	208	43	RCL
	209	11	11
	210	65	×
	211	53	(
	212	43	RCL
	213	07	07
	214	55	÷
	215	43	RCL
	216	02	02
	217	54	)
	218	45	Y^x
	219	43	RCL
	220	13	13
	221	95	=
	222	99	PRT
	223	43	RCL
	224	03	03
Go to E' if not in free stream flow	225	44	SUM
	226	07	07
	227	43	RCL
	228	10	10
	229	65	×
	230	43	RCL
	231	11	11
	232	95	=
	233	22	INV
	234	67	EQ
	235	10	E'
Stop	236	91	R/S

Description	Step	Code	Key
Subroutine A' to calculate u	237	76	LBL
	238	16	A'
Calculate r/r_2 from equation (3-86) and store in location 12	239	53	(
	240	43	RCL
	241	09	09
	242	33	X^2
	243	85	+
	244	43	RCL
	245	08	08
	246	33	X^2
	247	54	)
	248	34	√X
	249	55	÷
	250	43	RCL
	251	04	04
	252	95	=
	253	42	STO
	254	12	12
If $r/r_2 < 1$, go to C'	255	22	INV
	256	77	GE
	257	18	C'
u = 1	258	01	1
Return	259	92	RTN
Calculate u frm equation (3-93)	260	76	LBL
	261	18	C'
	262	01	1
	263	75	-
	264	43	RCL
	265	06	06
	266	65	×
	267	53	(
	268	01	1
	269	75	-
	270	43	RCL
	271	12	12
	272	45	Y^x
	273	01	1
	274	93	.
	275	05	5
	276	54	)
	277	33	X^2
	278	95	=
Return	279	92	RTN

Table C-3

OUTPUT OF WIND SPEED PROFILE PROGRAM FOR TI-59

Description	Output	Symbol
Input value of m	2.5	m
Input value of h_z	1.35	h_a
Input value of γ	0.2	γ
Input value of r_2	3.	r_2
Input value of z for lateral wind speed	1.35	z
First four points of lateral wind speed profile	0.	y
	.6585315158	U
	0.1	y
	.6626453063	U
	0.2	y
	.6701707758	U
	0.3	y
	.6798558844	U
First four points of vertical wind speed profile	0.	z
	0.	U
	0.1	z
	.4149507003	U
	0.2	z
	.4758552262	U
	0.3	z
	.5146331667	U

Table C-4

STORAGE LOCATIONS USED FOR WIND SPEED PROFILE PROGRAM FOR TI-59 CALCULATOR

Storage location	Symbol	Definition
01	m	Initial wind speed ratio
02	h_a	Height of turbine rotor hub in rotor radii
03	Δy	Interval between successive points in the profile calculation, in rotor radii
04	r_2	Wake radius in rotor radii
05	r_0	Initial wake radius in rotor radii
06	Δu_c	Wind speed deficit parameter
07	z	Altitude of wind speed profile
08	y	Lateral coordinate
09	z_v or z_v*	Altitude of profile relative to hub of real or image turbine
10	u	Wind speed in wake for wake of real turbine
11	u*	Wind speed in wake for wake of image turbine
12	r/r_2	Radial coordinate in wake normalized by wake radius
13	γ	Power law exponent for free stream wind speed profile

Appendix D

ALTERNATIVE ALGORITHMS FOR CALCULATION OF THE DOWNWIND EXTENT OF REGION I

In the development of the wake model, seven different approaches for the calculation of the downwind extent of Region I were considered. All of the approaches are based on the Abramovich approach for a jet in the absence of ambient turbulence. All of the approaches use the method of the square root of the sum of the squares for the adding of mechanical turbulence and ambient turbulence. For all of the approaches, the wake radius at the end of Region I is given by Abramovich equation (5.19) as

$$r_{21} = \frac{R_{21}}{R_0}\frac{R_0}{R_d} = \frac{r_0}{\sqrt{0.214+0.144m}} \qquad \text{(D-1)}$$

This equation was given as equation (3-34) in the description of the analytic model. It is the result of a momentum balance between the initial wake and the end of Region I; therefore, it should be valid regardless of the presence or absence of ambient turbulence.

* For mechanical turbulence, the downwind extent of Region I is given by Abramovich equation (5.20) as

$$(x_H)_m = \frac{1+m}{0.27(m-1)\sqrt{0.214+0.144m}} \qquad \text{(D-2)}$$

This equation was given as equation (3-35) in the description of the analytic model.

Definition of Approaches

For the purposes of identification, let the seven approaches for calculating the downwind extent of Region I be defined as follows.

1. r1 approach. - Add the effect of wake growth due to ambient turbulence to the erosion of the inner core as given by Abramovich, and define the end of Region I as the point at which the radius of the inner core becomes zero.

2. r2 approach. - Add the effect of wake growth due to ambient turbulence to the growth of the wake radius as given by Abramovich, and define the end of Region I as the point at which the radius of the wake becomes r_{21}.

3. b approach I. - Add the effect of wake growth due to ambient turbulence to the boundary layer growth given by Abramovich, and define the end of Region I as the point at which the width of the boundary layer becomes r_{21}. The width of the boundary layer is r_2-r_1.

4. b approach II. - Add twice the effect of wake growth due to ambient turbulence to the boundary layer growth given by Abramovich, and define the end of Region I as the point at which the width of the boundary layer becomes r_{21}. Twice the effect of wake growth due to ambient turbulence is added because ambient turbulence affects the boundary layer on both sides--at the potential core and at the wake boundary.

5. Streamline r1 approach. - This is the same as approach 1, except that erosion of the inner core is calculated relative to the streamline which passes through the initial wake radius, rather than relative to the line $r = r_0$ as was done for the r_1 approach.

6. Streamline r2 approach. - This is the same as approach 2, except that growth of the wake radius is calculated relative to the streamline which passes through the initial wake radius, rather than relative to the line $r = r_0$ as was done for the r_2 approach.

7. .5r21 approach. - This is the same as the first and second approaches, except that the erosion of the inner core and the growth of

the wake radius are calculated relative to a line passing from the initial wake radius at $x = 0$ to $r = .5r_{21}$ at the end of Region I. This line bisects the boundary layer. Mathematically, this approach is identical with approach 4.

All of these approaches take an angle associated with the Abramovich solution for the initial region, augment the wake growth rate for α, the wake growth rate due to ambient turbulence (using the square root of the sum of the squares for adding the turbulence components), and calculate the downwind extent of Region I accordingly. The difference in the approaches is in the choice of angles from the Abramovich solution. The choice of angle for the various approaches is shown in Figure D-1.

Description of Approaches

By definition, the end of Region I is that point at which the potential core vanishes. Thus, the wind speed at the center of the wake at the end of Region I is U_0. A momentum balance between the initial wake and the end of Region I gives equation (D-1).

From the equation for boundary layer growth given by his equation (5.1), Abramovich gives the length of the initial region of the wake as equation (D-2). The m subscript denotes that the quantity is associated with mecahically-generated turbulence. Therefore, for mechanically-generated turbulence, the slope of the radius of the inner core is (cf. Figure D-1)

$$\left(\frac{dr_1}{dx}\right)_m = \frac{-R_0}{(X_H)_m} = -r_0/(x_H)_m \qquad \text{(D-3)}$$

and the slope of the outer radius of the wake is

$$\left(\frac{dr_2}{dx}\right)_m = \frac{R_{21}-R_0}{(X_H)_m} = \frac{r_{21}-r_0}{(x_H)_m} \qquad \text{(D-4)}$$

Equation (D-1) which gives the wake radius when the potential core has been completely eroded is based upon a momentum balance (based upon the fact that $U = U_0$ at the center of the wake at the end of Region I and the assumption of the wind speed profile given by equation (3-32)) and is therefore valid regardless of the presence or absence of ambient turbulence. Therefore, the presence of ambient turbulence only affects the distance, X_H, at which the end of Region I occurs.

r_1 approach. - The first approach for calculating the value of x_H is the calculation of the effect of ambient turbulence on the erosion of the inner core and specifying the downwind extent of Region I as the point at which the radius of the inner core becomes zero. Under this definition the rate of erosion of the inner core is

$$\frac{dr_1}{dx} = -\left[\left(\frac{dr_1}{dx}\right)_m^2 + \alpha^2\right]^{\frac{1}{2}} \tag{D-5}$$

Since r_1 decreases for r_0 at $x = 0$ to 0 at $x = x_H$, the downwind extent of Region I is given by

$$x_H = \frac{r_0}{(dr_1/dx)} \tag{D-6}$$

where x_H is the downwind extent of Region I normalized by R_d.

r_2 approach. - The second approach for calculating x_H consists of calculation of the effect of ambient turbulence on the growth of the outer radius of the wake and specifying the downwind extent of Region I as the point at which the wake radius reaches the radius defined by a momentum deficit balance as given in equation (D-1). Under this definition,

$$\frac{dr_2}{dx} = \left[\left(\frac{dr_2}{dx}\right)_m^2 + \alpha^2\right]^{\frac{1}{2}} \tag{D-7}$$

Since r_2 increases from r_0 at $x = 0$ to r_{21} at $x = x_H$, the downwind extent of Region I is given by

$$x_H = \frac{r_{21} - r_0}{(dr_2/dx)} \tag{D-8}$$

b approach I. - The third approach of calculating x_H considers the growth of the boundary layer, r_2-r_1, and applies the wake growth due to ambient turbulence to the boundary layer. For mechanical turbulence, the growth rate of the boundary layer is

$$\left(\frac{db}{dx}\right)_m = \frac{r_{21}}{(x_H)_m} \qquad \text{(D-9)}$$

With ambient turbulence, the growth of the boundary layer is

$$\frac{db}{dx} = \left[\left(\frac{r_{21}}{(x_H)_m}\right)^2 + \alpha^2\right]^{\frac{1}{2}} \qquad \text{(D-10)}$$

Since the boundary layer grows from 0 at x = 0 to r_{21} at $x = x_H$, the downwind extent of Region I is

$$x_H = \frac{r_{21}}{db/dx} \qquad \text{(D-11)}$$

b approach II. - The fourth approach for calculating x_H uses the same criterion as the third approach, but augments the Abramovich solution for 2α, since the ambient turbulence affects both sides of the boundary layer (i.e., ambient turbulence acts to decrease the radius of the inner core and to increase the outer wake radius). Therefore, for this approach

$$\frac{db}{dx} = \left[\left(\frac{r_{21}}{(x_H)_m}\right)^2 + (2\alpha)^2\right]^{\frac{1}{2}} \qquad \text{(D-12)}$$

and

$$x_H = \frac{r_{21}}{db/dx} \qquad \text{(D-13)}$$

Streamline r_1 approach. - Figure D-2a shows Region I for the Abramovich solution. The three areas shown are the free stream flow, the potential core, and the boundary layer between the free stream flow and the potential core. Also shown is the streamline which passes through the initial wake boundary. It is assumed that the boundary layer develops on both sides of this streamline. In order to calculate the growth of the boundary layer in Region I, it is necessary to calculate the value of r_s, the radius of the streamline which passes through the initial wake radius at the end of Region I. The radius of the streamline, r_s, is determined by conservation of mass between the initial wake and the end of Region I. Therefore,

$$\pi r_0{}^2 U_0 = 2\pi \int_0^{r_s} U(r) r \, dr \qquad \text{(D-14)}$$

Since $U_0 = U_\infty/m$, the equation is written as

$$\frac{r_0{}^2 U_\infty}{m} = 2\int_0^{r_s} rU(r) \, dr \qquad \text{(D-15)}$$

The wind speed profile in Region I is given by equations (3-32) and (3-33). Multiplying equation (3-32) by $(U_0-U_\infty)/U_\infty$ and using the definition of m given in equation (3-9) gives

$$\frac{1}{m} = \frac{U}{U_\infty} = \left(\frac{1}{m} - 1\right)(1-\eta^{1.5})^2 \qquad \text{(D-16)}$$

or

$$U = U_\infty \left[\frac{1}{m} + \frac{m-1}{m} (1-\eta^{1.5})^2 \right] \qquad \text{(D-17)}$$

From equation (3-33), recalling that $r_1 = 0$ at the end of Region I (since the end of Region I is defined as that point at which the potential core vanishes) and $r_2 = r_{21}$ at the end of Region I,

$$r = r_{21}(1-\eta) \qquad \text{(D-18)}$$

and

$$dr = -r_{21}\, d\eta \qquad \text{(D-19)}$$

Using equations (D-17), (D-18), and (D-19) in equation (D-15) gives

$$\frac{r_0{}^2 U_\infty}{m} = \frac{-2U_\infty r_{21}{}^2}{m} \int_1^{\eta_s} (1-\eta)[1+(m-1)(1-\eta^{1.5})^2]d\eta \qquad \text{(D-20)}$$

or

$$\left(\frac{r_0}{r_{21}}\right)^2 = 2\int_{\eta_s}^{1}(1-\eta)d\eta + 2\int_{\eta_s}^{1}(m-1)(1-\eta)(1-2\eta^{1.5}+\eta^3)d\eta \qquad \text{(D-21)}$$

Integrating gives

$$\left(\frac{r_0}{r_{21}}\right)^2 = 2(\eta - \frac{\eta^2}{2})\Bigg|_{\eta_s}^{1} + 2(m-1)\left[\eta - \frac{\eta^2}{2} - \frac{2\eta^{2.5}}{2.5} + \frac{2\eta^{3.5}}{3.5} + \frac{\eta^4}{4} - \frac{\eta^5}{5}\right]\Bigg|_{\eta_s}^{1} \qquad \text{(D-22)}$$

Since (r_0/r_{21}) is a function of m only, η_s is a function of m only.

Equation (D-22) was solved numerically for four values of m as listed below.

m	η_s
1.5	0.390
2.0	0.383
2.5	0.378
3.0	0.375

A value of $\eta_s = 0.38$ is reasonable for the range of values of m for wind turbines. From equation (D-18)

$$r_s = 0.62r_{21} \qquad \text{(D-23)}$$

It is assumed that the wind speed profile given by equation (3-32) is valid in the presence of ambient turbulence. With this assumption, the relationship of equation (D-23) is valid in the presence of ambient turbulence. Figure D-2b shows wake growth in the presence of ambient turbulence.

Let β_1 be the distance from the streamline which passes through the initial wake boundary to the line between the boundary layer and the potential core. Then from Figure D-2a, the growth rate of β_1 due to mechanical turbulence (i.e., the Abramovich solution) is

$$\left(\frac{d\beta_1}{dx}\right)_m = \frac{r_s}{(x_H)_m} \tag{D-24}$$

where $(x_H)_m$ is given by equation (D-2).

The wake growth rate due to ambient turbulence is α. Adding the ambient turbulence to the mechanical turbulence by the square root of the sum of the squares of the ambient turbulence and the mechanical turbulence gives

$$\frac{d\beta_1}{dx} = \left[\left(\frac{r_s}{(x_H)_m}\right)^2 + \alpha^2\right]^{\frac{1}{2}} \tag{D-25}$$

Since $\beta_1 = r_s$ at the end of Region I, the downwind extent of Region I is

$$x_H = \frac{0.62 r_{21}}{\left[\left(\dfrac{0.62 r_{21}}{(x_H)_m}\right)^2 + \alpha^2\right]^{\frac{1}{2}}} \tag{D-26}$$

<u>Streamline r_2 approach</u>. - For the sixth approach for calculating the downwind extent of Region I, the portion of the boundary layer above the streamline which passes through the initial wake boundary is considered. Let β_2 be the distance from the streamline which passes through the initial wake boundary to the line between the boundary layer and the free stream. Then from Figure D-2a, the growth in β_2 due to mechanical turbulence is

$$\left(\frac{d\beta_2}{dx}\right)_m = \frac{r_{21}-r_s}{(x_H)_m} \tag{D-27}$$

Adding ambient turbulence in the manner described above gives

$$\frac{d\beta_2}{dx} = \left[\left(\frac{r_{21}-r_s}{(x_H)_m}\right)^2 + \alpha^2\right]^{1/2} \tag{D-28}$$

Since

$$\beta_2 = r_{21}-r_s = 0.38r_{21} \tag{D-29}$$

the downwind extent of Region I is

$$x_H = \frac{0.38r_{21}}{\left[\left(\frac{0.38r_{21}}{(x_H)_m}\right)^2 + \alpha^2\right]^{1/2}} \tag{D-30}$$

$0.5r_{21}$ approach. - For the seventh approach for calculating the downwind extent of Region I, the boundary layer is assumed to develop about a line which bisects the boundary layer. The boundary layer is assumed to develop equally on both of its sides. In this case

$$r_s = 0.5r_{21} \tag{D-31}$$

In this case, the line connecting the initial wake boundary and r_s at the end of Region I is not a streamline. The downwind extent of Region I is developed in the same manner as presented above for the previous two approaches. The results are identical for development based on β_1 or on β_2 above because in this case $\beta_1 = \beta_2$. For both cases,

$$x_H = \frac{0.5r_{21}}{\left[\left(\frac{0.5r_{21}}{(x_H)_m}\right)^2 + \alpha^2\right]^{1/2}} \tag{D-32}$$

It is noted that multiplication of the numerator and denominator of the right side of equation (D-32) gives the identical result given by equations (D-12) and (D-13). Therefore, the b approach II and the $0.5r_{21}$ approach are mathematically identical.

Comparison of Results

Figure D-3 shows the donwwind extent of Region I as a function of α, the wake growth rate due to ambient turbulence, for the first four approaches presented above. For Figure D-3, m = 3. Figure D-4 shows similar data for the last three approaches for calculating the downwind extent of Region I. For Figure D-4, m = 3. Figures D-5 and D-6 show information similar to that shown in Figures D-3 and D-4, respectively, but for m = 2. As mentioned previously, approaches 4 and 7 are mathematically identical.

Approaches 3 and 4 are identical, except that approach 3 augments the Abramovich solution for α, whereas approach 4 augments the Abramovich solution for 2α. From a physical point of view, the 2α approach is more reasonable than the α approach because ambient turbulence affects the growth of the boundary layer on both sides of the boundary layer.

In Region I, mass continuity demands that the flow have an inward radial component. This is illustrated by the fact that the streamline which passes through the intital wake radius has an inward component. For approaches 1 and 2, the wake growth rate is taken relative to the line $r = r_0$, which has no physical relationship to the flow direction or to the boundary layer. Relative to the flow and to the boundary layer, the line $r = r_0$ is a randomly drawn line. From a physical point of view, the measurement of the growth of the boundary layer from the streamline or from the line which bisects the boundary layer is more plausible.

For these reasons, from a physical point of view, approaches 4, 5, 6, and 7 are the most plausible approaches for the calculation of the downwind extent of Region I. Figures D-4 and D-6 show that the results do not vary greatly between these approaches (recalling that approach 4 is mathematically identical to approach 7). For these reasons, approaches 4 and 7 were chosen as the approaches for calculating the downwind extent of Region I in the model.

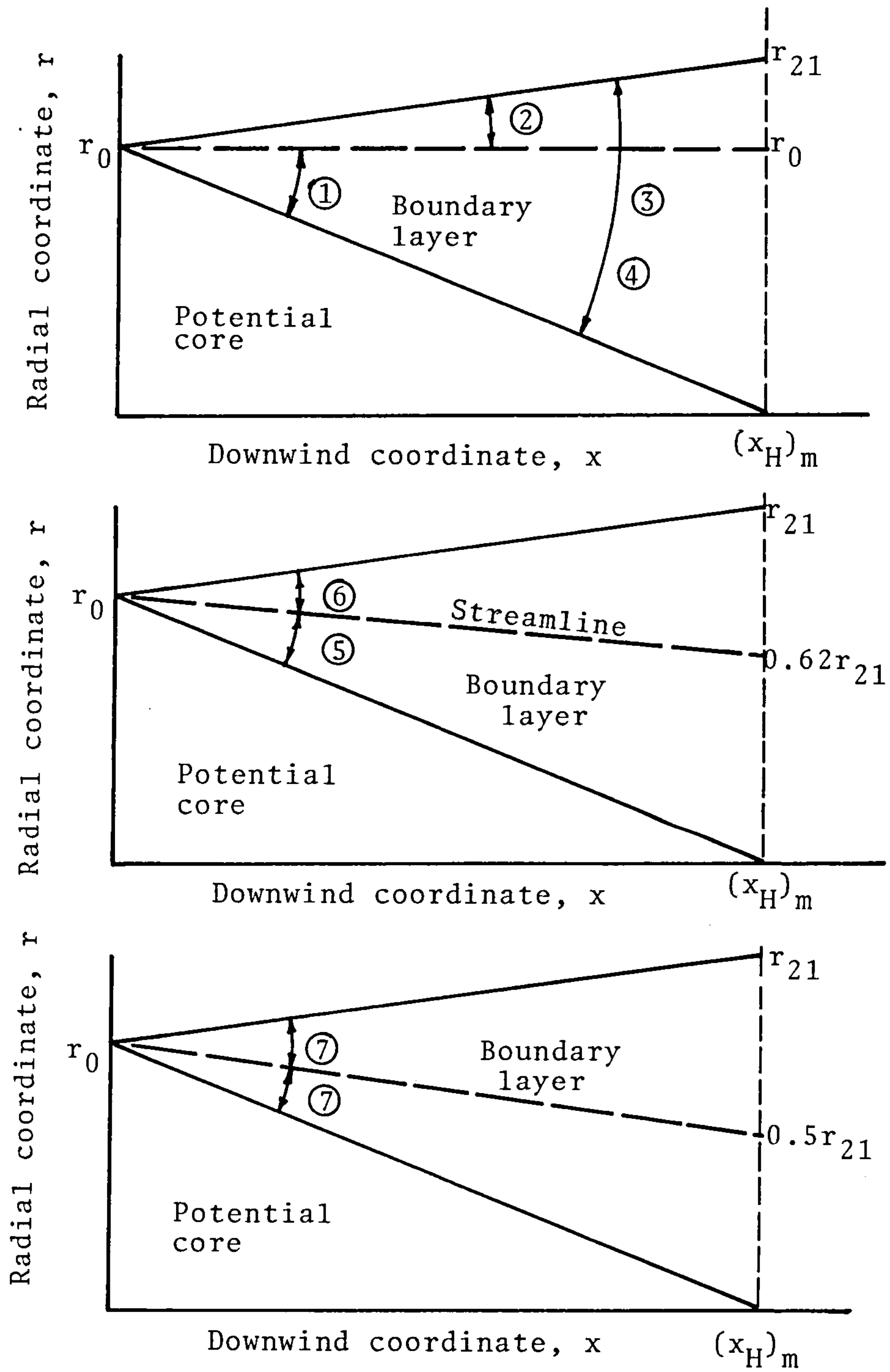
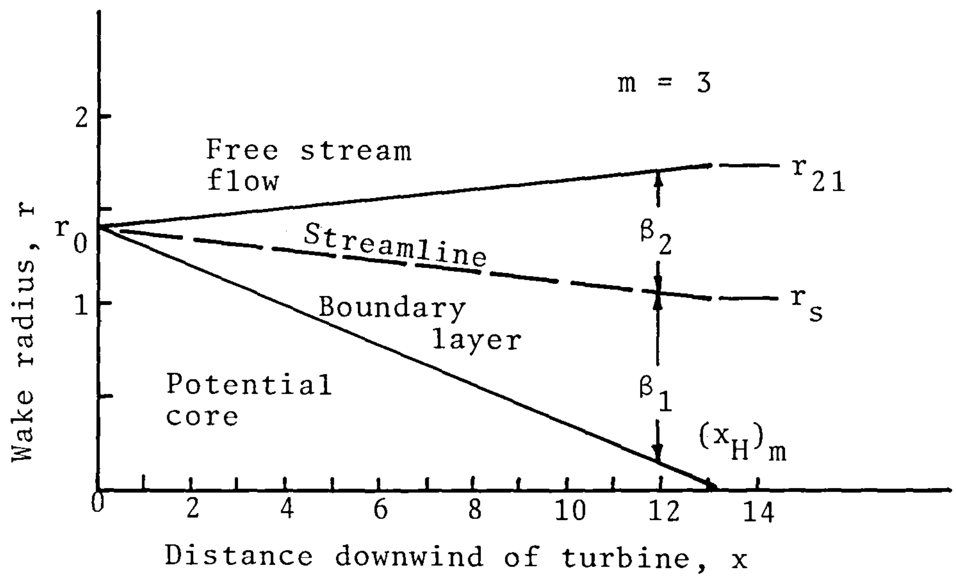
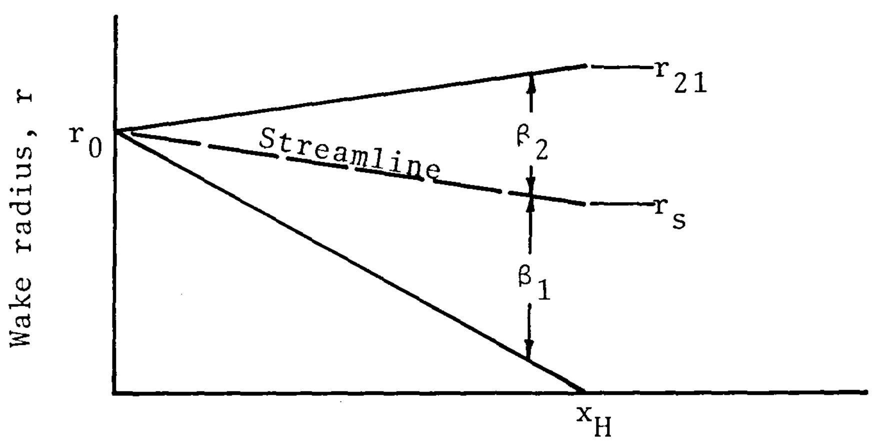


Figure D-1. - Angle used from Abramovich model to be augmented for ambient turbulence and used to calculate downwind extent of Region I



(a) Region I for Abramovich solution (no ambient turbulence)



(b) Region I with ambient turbulence

Figure D-2. Geometry of Region I of the wake.

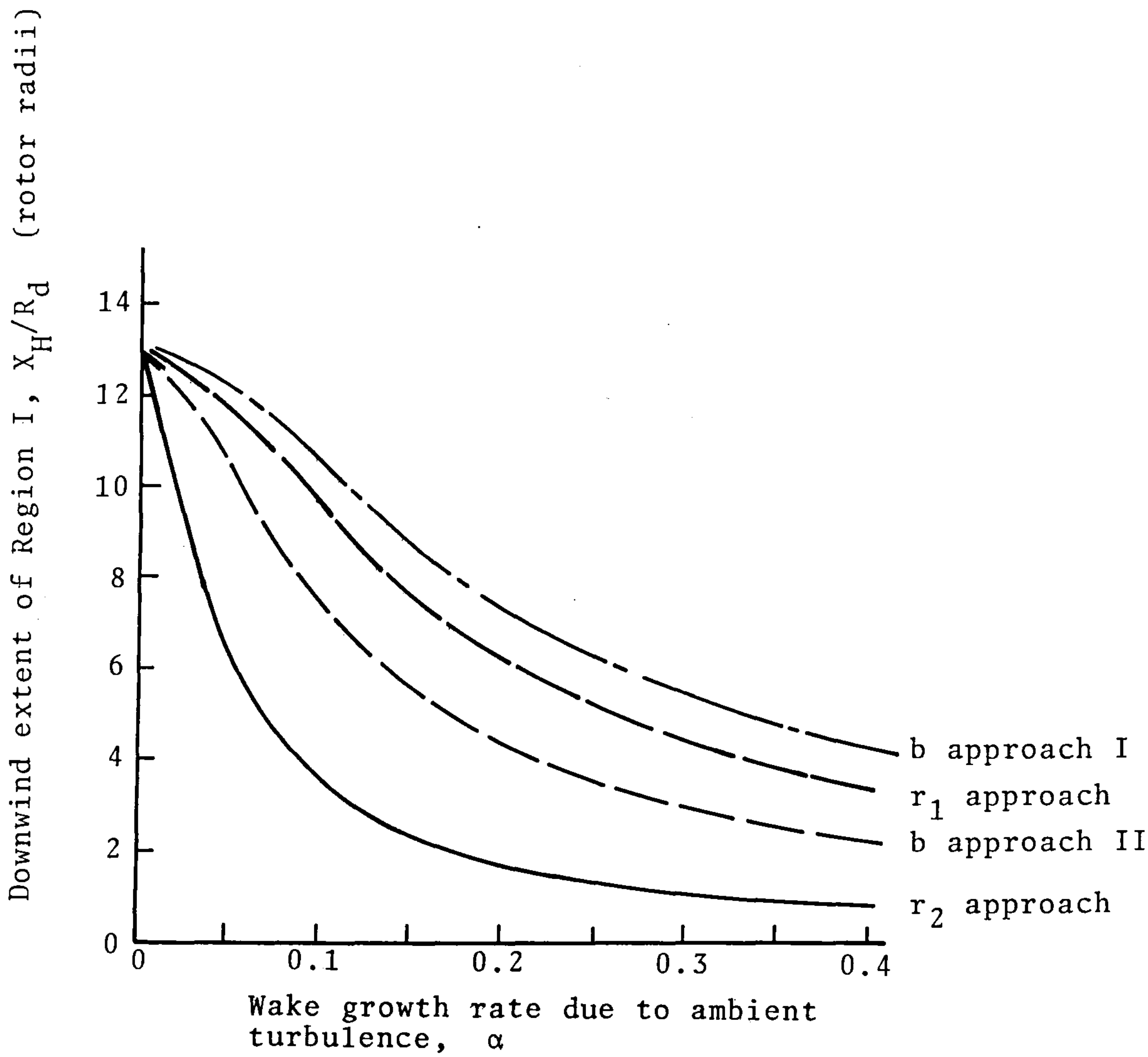


Figure D-3. - Comparison of first four approaches for calculating x_H for m = 3.

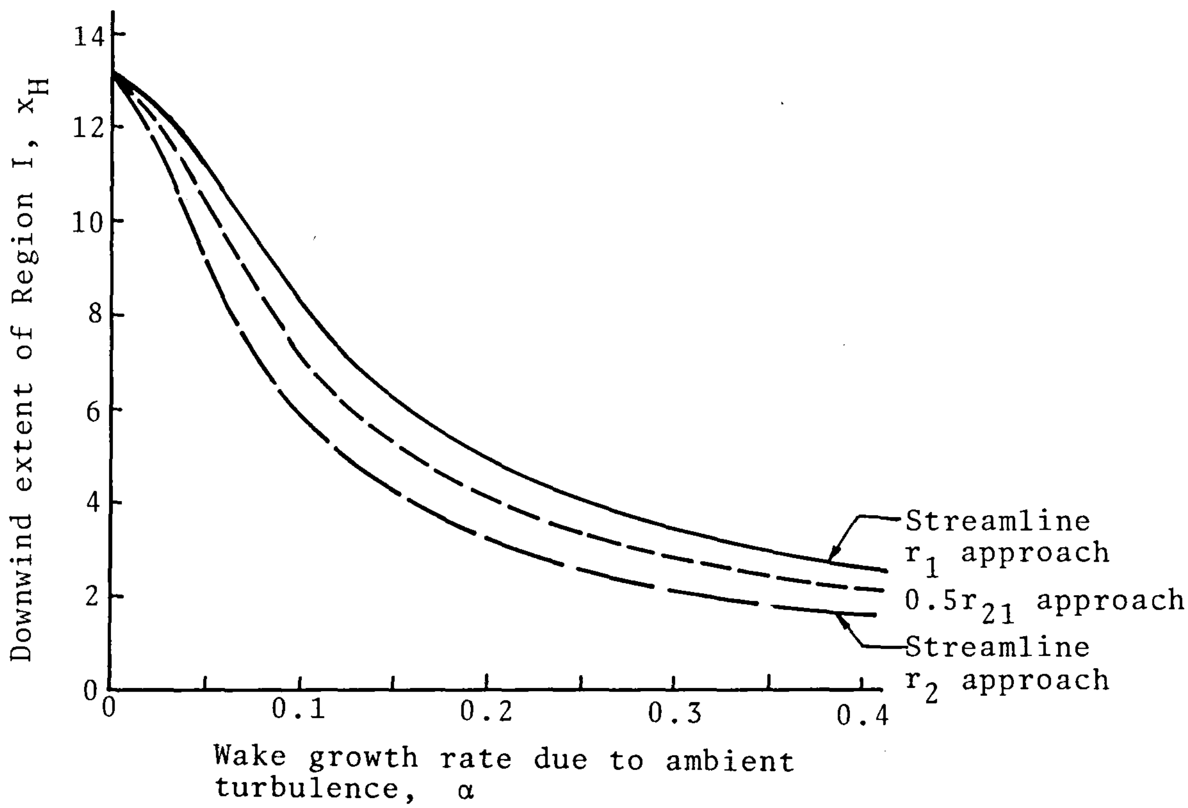


Figure D-4. - Comparison of last three approaches for calculating x_H for m = 3.

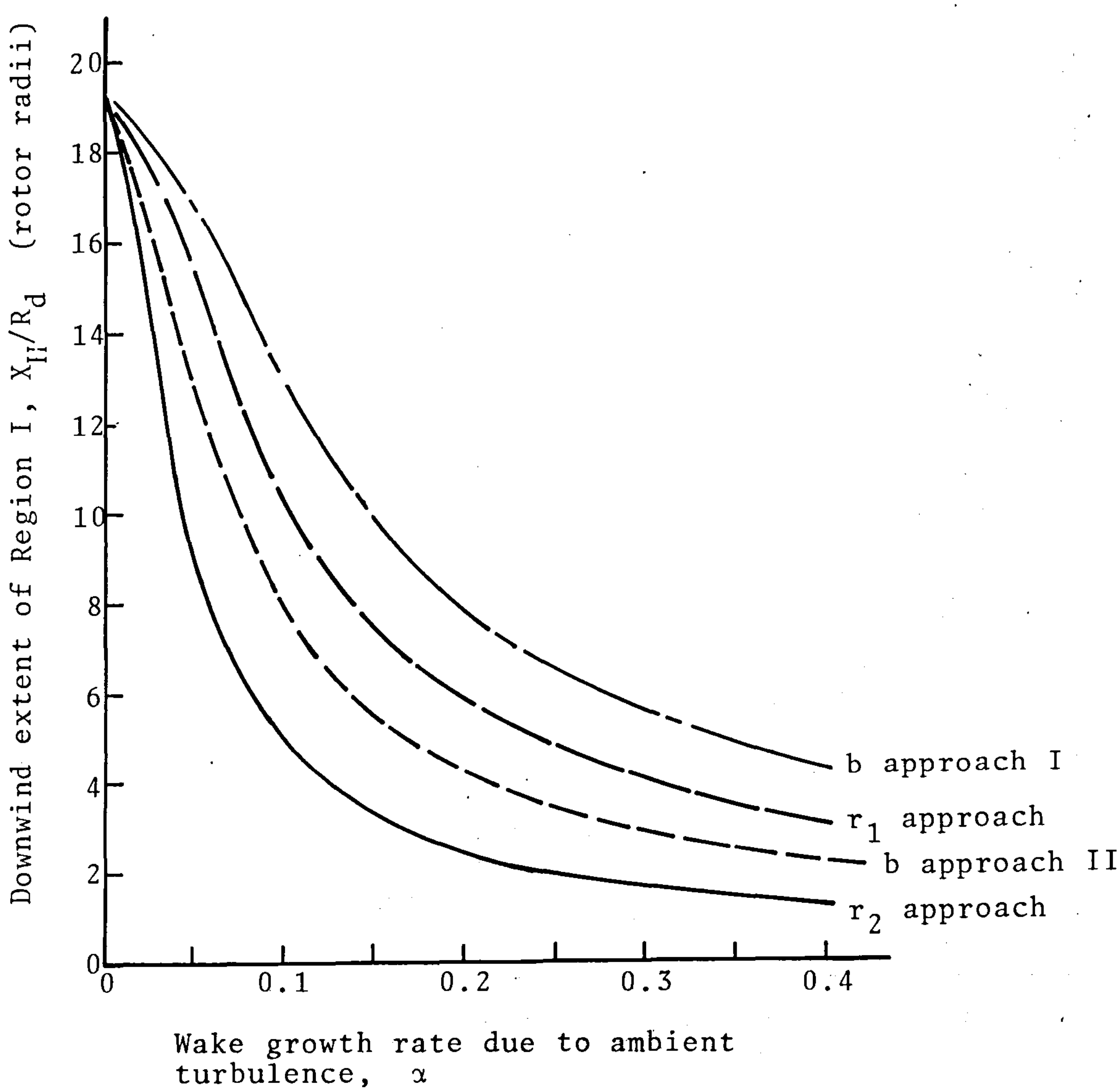


Figure D-5. - Comparison of first four approaches for calculating x_H for m = 2.

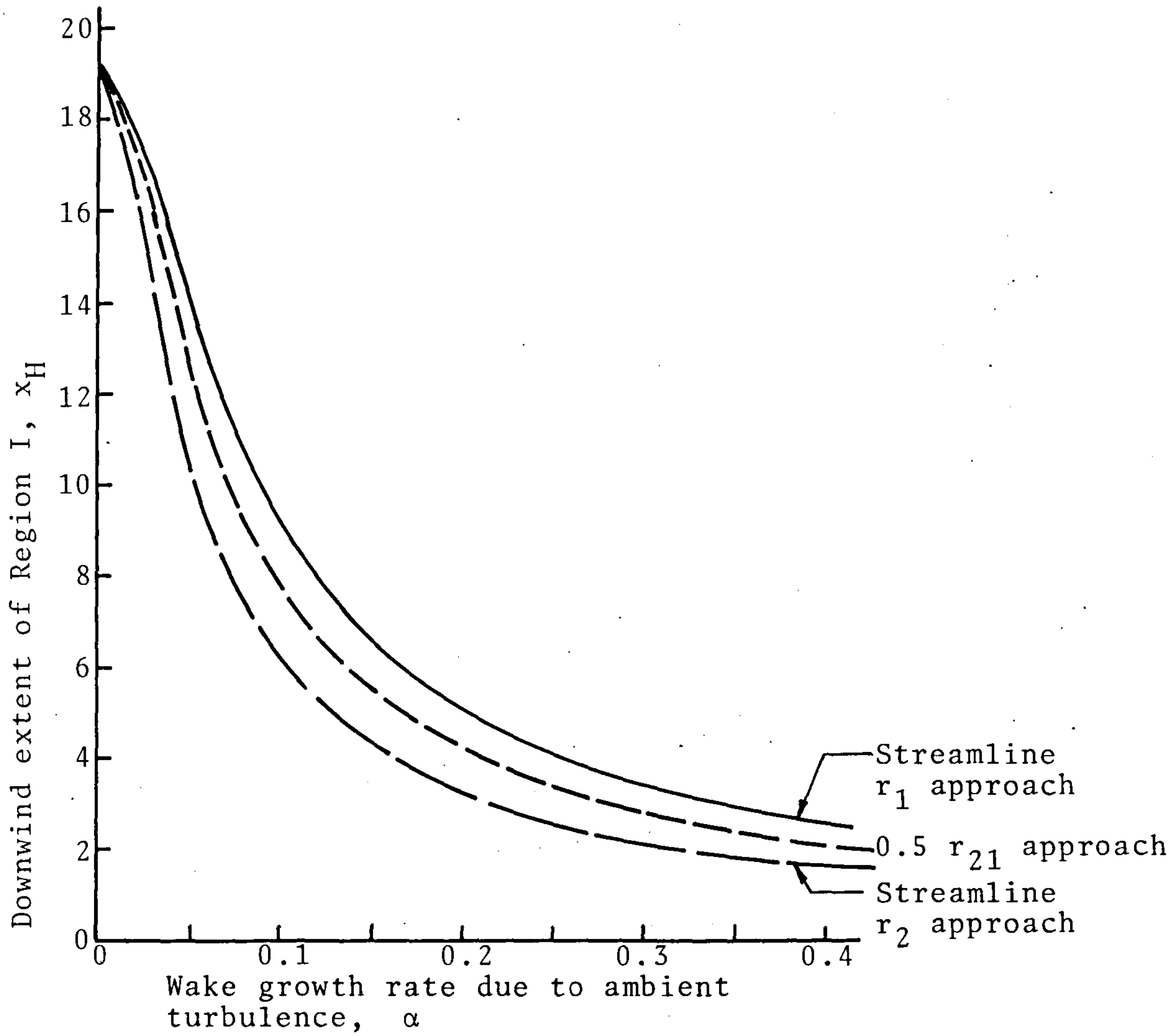


Figure D-6. - Comparison of last three approaches for calculating x_H for m = 2.

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16. Abstract

As part of the DOE/NASA research program on wind energy, a computer program to calculate the wake downwind of a wind turbine was developed. Turbine wake characteristics are useful for determining optimum arrays for wind turbine farms. The analytical model is based on the characteristics of a turbulent coflowing jet with modification for the effects of atmospheric turbulence. The program calculates overall wake characteristics, wind profiles, and power recovery for a wind turbine directly in the wake of another turbine, as functions of distance downwind of the turbine. The calculation procedure is described in detail, and sample results are presented to illustrate the general behavior of the wake and the effects of principal input parameters.

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